APPENDIX 1

EQUATIONS FOR THE MICHIGAN BROWN-DEARDORFF-STERN (BDS) CGE TRADE MODEL

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COUNTRY EQUATIONS

A. Final Demand

(1) \( \hat{C}^T_{ij} = \hat{E}_i - \hat{P}^T_{ij} \) \( j = 1..n \)

(2) \( \hat{C}^N_{ij} = \hat{E}_i - \hat{P}^N_{ij} \) \( j = n + 1..n' \)

B. Intermediate Demand

(3) \( \hat{Z}^T_{ijk} = \hat{S}_{ik} \) \( j = 1..n \) \( k = 1..n' \)

(4) \( \hat{Z}^N_{ijk} = \hat{S}_{ik} \) \( j = n + 1..n' \) \( k = 1..n' \)

C. Total Demand

(5) \( \hat{D}^T_{ij} = v_{ijo} \hat{C}^T_{ij} + \sum_{k=1}^{n'} v_{ijk} \hat{Z}^T_{ijk} \) \( j = 1..n \)

(6) \( \hat{D}^N_{ij} = v_{ijo} \hat{C}^N_{ij} + \sum_{k=1}^{n'} v_{ijk} \hat{Z}^N_{ijk} \) \( j = n + 1..n' \)

D. Product Demand

(7) \( \hat{D}^m_{ij} = \hat{D}^T_{ij} + \sigma_{ij} \theta_{ij} (\hat{P}^T_{ij} - \hat{P}^m_{ij}) - \frac{\sigma_{ij} \theta_{ij}}{\sigma_{ij} - 1} \hat{n}_{ij} \) \( j = 1..n \)

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\[
\hat{D}_{ij} = \hat{D}_{ij}^{m} + \sigma_j (\hat{P}_{ij}^{m} - \hat{P}_{ij}^{r}) \quad j = 1..n
\]

\[
\hat{D}_{ij}^{E} = \hat{D}_{ij}^{m} + \sigma_j (\hat{P}_{ij}^{m} - \hat{P}_{ij}^{E}) \quad j = 1..n
\]

\[
\hat{D}_{ij}^{m} = \hat{D}_{ij}^{r} + \sigma_j \theta_j^{m} (\hat{P}_{ij}^{m} - \hat{P}_{ij}^{*}) - \frac{\sigma_j \theta_j}{\sigma_j - 1} \hat{n}_{ij} \quad j = 1..n
\]

E. Prices

\[
\hat{p}_{ij}^{r} = \theta_j^{r} \hat{p}_{ij}^{r} + \theta_j^{m} \hat{p}_{ij}^{m} - \frac{\theta_j}{\sigma_j - 1} \hat{n}_{ij} \quad j = 1..n
\]

\[
\hat{p}_{ij}^{m} = \sum_{r=1}^{m+1} \theta_j^{m} \hat{p}_{ij}^{r} - \sum_{r=1}^{m+1} \frac{\theta_j}{\sigma_j} \hat{n}_{ij} \quad j = 1..n
\]

\[
\hat{p}_{ij}^{r} = \hat{p}_{ij}^{r} + \hat{r}_{ij} \quad r = 1..(m+1)\]

\[
\hat{p}_{ij}^{m} = \hat{p}_{ij}^{m} \quad j = 1..n
\]

\[
\hat{p}_{ij}^{n} = M\hat{C}_{ij} \quad j = n+1..n'
\]

\[
\hat{p}_{ij}^{*} = \theta_j^{MC} \hat{M} C_{ij} + \theta_j^{FC} \left[ \hat{p}_{ij}^{v} + \hat{n}_{ij} - \hat{S}_{ij} \right] \quad j = 1..n
\]

\[
\hat{p}_{ij}^{v} = \frac{\hat{n}_{ij}}{\eta_{ij} + 1} \quad j = 1..n
\]

\[
\hat{p}_{ij}^{*} = \theta_j^{L} \hat{w}_{ij} + \theta_j^{C} \hat{\gamma}_i \quad j = 1..n'
\]

F. Marginal Cost

\[
M\hat{C}_{ij} = b_{ij} \theta_j^{\text{MK}} \hat{P}_{ij}^{y} + \sum_{k=1}^{n'} b_{ik} \theta_j^{\text{MC}} \hat{P}_{ik} \quad j = 1..n'
\]
G. Demand for Primary Inputs

\( \hat{V}_j = \hat{S}_j - \hat{n}_{ij} \quad j = 1..n' \)  

\( \hat{L}_j = \theta_{ij}^{\nu} \hat{V}_j - \overline{\sigma}_j \theta_{ij}^{\kappa}(\hat{w}_j - \hat{r}_j) + \hat{n}_{ij} \quad j = 1..n' \)

\( \hat{K}_j = \theta_{ij}^{\nu} \hat{V}_j + \overline{\sigma}_j \theta_{ij}^{\lambda}(\hat{w}_j - \hat{r}_j) + \hat{n}_{ij} \quad j = 1..n' \)

H. Non-tradable Goods Market Equilibrium

\( \hat{S}_{ij}^N = \hat{D}_{ij}^N \quad j = n + 1..n' \)

I. Demand Elasticities

\( \hat{n}_j = \frac{\sigma_{ij-1}}{n_{ij} n_{ij}} \theta_{ij}(\hat{D}_j + \hat{P}_j - \hat{P}_j^T - \hat{D}_j^T) \quad (r / i = 1,...m, r \neq i, j = 2..n) \)

\( \hat{n}_j = \frac{\sigma_{ij-1}}{n_{ij} n_{ij}} \theta_{ij}(\hat{D}_j + \hat{P}_j - \hat{P}_j^T - \hat{D}_j^T) \quad (j = 2..n, i = 1,...m) \)

\( \hat{n}_j = \sum_{j=1}^{m} \delta_{ij} \hat{n}_{ij} \quad (i = 1,...m, j = 2..n) \)

J. Primary Factors Market Equilibrium

\( \sum_{j=1}^{n'} h_{ij} \hat{k}_j = 0 \quad (for \ i = 1,...m) \)

\( \sum_{j=1}^{n'} h_{ij} \hat{L}_j = 0 \quad (for \ i = 1,...m) \)

K. National Income Determination

\( d B^i_t = d B^i_{t0} \)
L. Nontariff Barriers

\[ \hat{r}_{ij}^{\text{Meq}} = \hat{r}_{ij} + (\hat{D}_{ij} - \hat{Q}_{ij} + \hat{n}_{ij}) \frac{\theta_{ij}^Q}{\sigma_{ij}(1 - \theta_{ij}^Q)} \quad j = 1, \ldots, n \]
\[ r \neq i; \quad r = 1, \ldots, m \]

**WORLD EQUATIONS**

A. Trade Balance

\[ dB_i^T = \sum_{r=1}^{m} X_{ri}^i (\hat{D}_{ri}^i + \hat{P}_{wi}^i) + \sum_{j=2}^{m} \sum_{r \neq i} X_{rij} (\hat{D}_{rij} + \hat{n}_{rij} + \hat{P}_{wrij}) + \sum_{j=1}^{n} X_{ij}^E (\hat{P}_{wij}^i + \hat{L}_{wij}) \]
\[ - \sum_{r=1}^{m} M_{ij} (\hat{D}_{rij}^r + \hat{P}_{wrij}^r) - \sum_{j=2}^{m} \sum_{r \neq i} M_{ij}^r (\hat{D}_{rij}^r + \hat{n}_{rij} + \hat{P}_{wrij}) - \sum_{j=1}^{n} M_{ij}^E (\hat{D}_{rij}^E + \hat{P}_{wij}) \]
\[ \text{(for } i = 1, \ldots, m) \]

B. Tradable Goods Market Equilibrium

(32A) \[ S_{ij}^T \hat{S}_{ij}^T = D_{ij}^E n_{ij} \left[ \hat{n}_{ij} + \hat{L}_{ij} \right] + \sum_{r=1}^{m} n_{ij} D_{ij}^r \left[ \hat{n}_{ij} + \hat{D}_{ij} \right] \quad j = 1, \ldots, n \]
\[ j \neq \text{agriculture} \]

(32B) \[ S_{ij}^T \hat{S}_{ij}^T = \sum D_{ij}^E \hat{D}_{ij}^E + D_{ij}^E \hat{L}_{ij} \quad \text{for } j = \text{agriculture} \]

(33) \[ dS_{ij}^E = \sum_{i=1}^{m} D_{ij}^E \hat{D}_{ij}^E \quad j = 1, \ldots, n \]

(34) \[ dS_{ij}^E = S_{ij}^E \hat{P}_{wij}^E \quad j = 1, \ldots, n \]

C. Row Import Licensing

(35) \[ O = \sum_{j=1}^{n} P_{wij}^E \left[ dS_{ij}^E + S_{ij}^E \hat{P}_{wij}^E \right] - \sum_{j=1}^{n} \left[ M_{ij}^E \hat{L}_{ij} + \sum_{i=1}^{m} M_{ij}^E \hat{P}_{wij}^i \right] \]
Variables

- $C_{ij}^T, C_{ij}^N$: Final demand for tradable and nontradable goods $j$ in country $i$.
- $E_i$: Household income in country $i$.
- $P_{ij}^T, P_{ij}^N$: Price index of tradable and nontradable goods $j$ in country $i$.
- $Z_{ijk}^T, Z_{ijk}^N$: Intermediate demand for tradable and nontradable goods $j$ by industry $k$ in country $i$.
- $S_k$: Production of good $k$ in country $i$.
- $D_{ij}^T, D_{ij}^N$: Total demand for tradable and nontradable goods $j$ in country $i$.
- $D_{ij}^M$: Total demand for imports of good $j$ in country $i$.
- $P_{ij}^M$: Price index of import good $j$ in country $i$.
- $P_{ij}^V$: Price of domestic good produced by a representative firm in industry $j$ in country $i$.
- $n_{ij}$: Number of firms in industry $j$ in country $i$.
- $D_{ij}^r$: Demand in country $i$ of the good produced by a representative firm in industry $j$ in country $r$.
- $t_{ij}^{Mreq}$: Tariff equivalent imposed by country $i$ on imports of good $j$ from country $r$.
- $P_w$: World price of good $j$ produced in country $i$.
- $MC_{ij}^T$: Marginal cost of a representative firm in country $i$ in industry $j$.
- $S_{ij}^T$: Tertiary demand for tradable and nontradable goods $j$ in country $i$.
- $v_{ij}$: Primary input aggregate demanded by a representative firm in industry $j$ in country $i$.
- $L_{ij}^T$: Demand for labor in industry $j$ in country $i$.
- $K_{ij}^T$: Demand for capital in industry $j$ in country $i$.
- $B_i^T$: Country $i$’s trade balance.
- $Q_{ij}^r$: Quota restriction on imports of good $j$ from country $r$ by country $i$.
- $L_e$: Import licensing variable for ROW.

Parameters

- $v_{ij}$: Final consumption share of total purchases of good $j$ in country $i$.
- $v_{ijk}$: Intermediate demand for good $j$ by industry $k$ share of purchases of good $j$ in country $i$.
- $\sigma_{ij}$: Elasticity of substitution between different varieties of good $j$ in country $i$.
- $\theta_{ij}^M$: Fraction of expenditure on good $j$ in country $i$ devoted to imports.
- $\theta_{ij}^r$: Fraction of expenditure on good $j$ in country $i$ devoted to goods produced in country $r$.
- $\theta_{ij}^{MC}$: Variable input share of total cost in industry $j$ in country $i$.
- $\theta_{ij}^{FC}$: Fixed cost share of total cost in industry $j$ in country $i$.  

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\( \theta^L_{ij} \) Labor’s share of expenditure on primary inputs in industry j in country i.

\( \theta^K_{ij} \) Capital’s share of expenditure on primary inputs in industry j in country i.

\( \theta^{VK}_{ij} \) Variable capital’s share of total capital employed in industry j in country i.

\( b_{ij} \) Primary input share of total cost in industry j in country i.

\( b_{ik} \) Intermediate input k’s share of total cost of production in industry j in country i.

\( \sigma_{ij} \) Elasticity of substitution between capital and labor in industry j in country i.

\( \delta^r_{ij} \) Fraction of sales by a representative firm in industry j in country i that go to country r.

\( h^K_{ij} \) Fraction of capital in country i employed in industry j.

\( h^L_{ij} \) Fraction of labor in country i employed in industry j.

\( K^i \) Capital stock in country i.

\( L^i \) Labor supply in country i.

\( B^i \) Trade balance of country i.

\( t'_{ij} \) Tariff imposed by country i on imports of good j from country r.

\( \theta^{QR}_{ij} \) Fraction of imports of good j by country i from country r that are subject to quantitative restrictions.

\( X^j \) Country i’s exports of good j.

\( X'^j \) Country i’s exports of good j to country r.

\( M^j \) Country i’s imports of good j from country r.

\( S^J \) ROW supply of good j.

\( \varepsilon^x_j \) ROW supply of good j price elasticity.
EXPLANATION OF THE EQUATIONS OF THE MICHIGAN BROWN-DEARDORFF-Stern (BDS) CGE TRADE MODEL

COUNTRY EQUATIONS

For each country, various supply and demand conditions must be specified:

A. Final Demand

Equations (1) and (2) give final demand for tradable and nontradable goods aggregates, respectively. The utility function at this level is Cobb-Douglas. Therefore, a constant fraction of the budget is spent on each good.

B. Intermediate Demand

Equations (3) and (4) give intermediate demand for tradable and nontradable input j by sector k, respectively. All intermediate inputs into production are used in fixed proportion to output. Therefore, if output in sector k rises by 5 percent then the use of every single intermediate input from j=1 to j=n’ must also rise by 5 percent in sector k.

C. Total Demand

Total demand for tradable and nontradable goods is given by equations (5) and (6), respectively. To obtain total demand we begin with household demand for good j. In addition, we must add in intermediate demand for good j by all of the k different sectors. In percent change terms, the total change in demand for good j is a weighted average of the percent change in final and intermediate demand for good j.

D. Product Demand

Total demand for good j is then broken down into the various import and domestic varieties of good j. First, total demand is allocated between the import aggregate (equation 7) and a typical domestically produced variety (equation 10). As can be seen from the first term of these two equations, the first determinant of import demand and domestic demand is total demand. Suppose that the number of firms and relative prices do not change. Then, if total demand rises by 2 percent, import and domestic demand will each also rise by 2 percent.

Of course, if relative prices do change then consumers will reallocate between imports and the domestic variety. If the price of the domestic variety rises relative to imports, then consumers will certainly substitute away from the domestic good toward imports, as can be seen from the second terms in each equation. How much substitution takes place depends on the elasticity of substitution, sigma. The higher sigma, the more sensitive consumers are to a change in relative prices.

Another force affecting demand for the import aggregate and each of the domestic varieties is the number of domestic firms. As the number of domestic firms rises, then demand for each individual incumbent domestic firm will fall and the demand for the import aggregate will fall, as can be seen from the last term in equations (7) and (10).
Demand for the import aggregate must then be allocated across competing firms in the world, as shown in equation (8) for the included trade partners and equation (9) for the rest of the world. As with domestic demand, an increase in the demand for the import aggregate will raise demand for each individual foreign variety. In addition, if the price of a typical foreign variety falls relative to other import prices, then demand for that variety will rise. Again, the sensitivity depends on the elasticity of substitution, sigma. A high value for sigma implies that the various varieties are close substitutes. Therefore, consumers will be very sensitive to price changes.

E. Prices

There are various price relationships that must hold in the model. Some are merely definitional. For example, equations (11) and (12) define the price indexes for tradable goods and imports, respectively. In each case, the price index is a market-share weighted average of the component prices. The constituent parts of the price of tradables are the price of the domestically produced variety and the import price aggregate. The import price aggregate is itself comprised of the prices of all of the individual imported varieties.

In both cases, notice that the number of varieties enters negatively. That is, as the number of varieties rises, the price index falls. The reasoning is: consumers like variety. The greater the number of firms the more variety available to consumers. Therefore, consumers can achieve a certain level of utility at a lower cost if more varieties are available.

The sensitivity of the price index to the number of varieties depends on the elasticity of substitution. The lower sigma the more the price index falls when new varieties are introduced. A low sigma implies that varieties are very different from one another. That is, adding a new variety brings a lot of additional value to the consumer. Thus, there is a large decline in the implicit cost of achieving a particular level of utility from consumption.

The next two equations, (13) and (14), link world prices to domestic prices. Equation (13) states that the landed price of imports from country r is determined by the world price and the tariff equivalent. Roughly speaking, if the world price rises by 10 percent and tariffs fall by 2 percent, then the landed price of imports will rise by 8 percent.

Equation (14) states that the domestic price of a typical domestic variety is equal to the price determined on world markets. This is the case because import tariffs are not applied to domestically produced goods.

The next two equations, (15) and (16), relate price and marginal or average total cost. All of the nontradable markets are taken as perfectly competitive. Therefore, the price of each nontradable must be equal to marginal cost, as can be seen from equation (15).

 Tradable goods, however, are produced under increasing returns to scale conditions. Nevertheless, free entry occurs, so that price must still equal total cost, as is given in equation (16).

Average total cost is made up of marginal cost, the first term in (16), and average fixed cost, the second term. To find the percent change in ATC, simply take a cost-share weighted average of the percent change in marginal and average fixed costs.

It is worth pointing out here something of the nature of technology. As will be seen below, marginal cost consists of the cost of labor, capital, and intermediate inputs. However, labor and capital are the only fixed inputs. No immediate inputs are required for fixed costs. The costs of fixed labor and capital inputs are captured by the first term in the square brackets in equation (16).
Average fixed cost also depends negatively on the output of a typical firm. Firm output is given by industry output divided by the number of firms in the industry. In percent terms, industry output is the difference between the percent change in industry output and the number of firms, as is given by the last two terms in equation (16).

Imperfectly competitive firms must also charge the optimal mark-up of price over marginal cost, as can be seen from equation (17). The optimal mark-up depends on the elasticity of demand. The higher the elasticity (in absolute value), the smaller the gap between price and marginal cost.

Finally, the price of value added, given by equation (18), is just a cost share weighted average of the cost of labor and the cost of capital.

F. Marginal Cost

There are two components of marginal cost, as can be seen by equation (19). The first term is the cost of primary inputs (capital and labor) and the second term is the intermediate inputs from all of the goods sectors.

G. Demand for Primary Inputs

Capital and labor are combined to form a primary input aggregate. The primary input aggregate is then used in fixed proportion to firm output. This can be seen in equation (20), where again we calculate the percent change in firm output as the difference between the percent change in industry output and the number of firms in the industry.

Once the demand for the primary input aggregate has been determined, expenditure is then allocated between capital and labor. Labor demand by industry, in equation (21), depends on the demand for the primary aggregate, the relative price of capital and labor and the number of firms in the industry. The industry demand for capital in equation (22) is similarly determined.

H. Nontradable Goods Market Equilibrium

Market clearing in the nontradable goods market simply requires that domestic demand equal domestic supply, as can be seen from equation (23).

I. Demand Elasticities

The demand elasticities are given in equations (24)-(26). The firm must first calculate the elasticity of demand for its product in each market, as in equations (24) and (25). The demand curve will be perceived as more elastic if the number of competing firms rises or the prices of competing varieties falls relative to the domestic variety.

Firms then aggregate price elasticities over the various markets that they serve to calculate a single price elasticity for their output. The aggregate elasticity is a market-share weighted average of the individual elasticities, as given by equation (26).

J. Primary Factors Market Equilibrium
Capital and labor markets clear when the sum over industry demand for each factor equals an exogenous supply. The capital and labor market equilibrium conditions are given in equations (27) and (28), respectively.

K. National Income Determination

In principle, household income is given by payments to primary factors, tariff revenue and international borrowing. However, due to Walras’ Law, this condition is equivalent to stating that the change in the trade balance is set equal to the exogenous trade balance, as is given by equation (29).

L. Nontariff Barriers

There are two types of trade barriers that affect imports, nominal tariffs and quotas. Each quota is captured by its tariff equivalent. The tariff equivalent is calculated endogenously in the model so that if demand or supply conditions change then the tariff equivalent will also change.

The actual tariff that applies is then composed of the nominal tariff (first term in equation (30)) and the tariff equivalent of the nontariff barrier (second term). The quota facility is invoked by specifying the fraction of the sector covered by nontariff barriers and the desired change in the quota limit.

WORLD EQUATIONS

Supply and demand conditions for each country are passed to the world system to determine equilibrium prices.

A. Trade Balance

The trade balance is a straightforward sum over the change in the value of exports less the changes in the value of imports.

B. Tradable Goods Market Equilibrium

The market for a representative variety in each country merely requires that total supply equal world demand for each variety. Similar conditions apply for ROW supply and demand.

C. ROW Import Licensing

Rest of world demand for imports is determined so that the trade balance is held constant. Therefore, any revenue earned through exports is then spent on imports.