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**Firm Entry, Trade, and Welfare in Zipf's World**

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# Firm Entry, Trade, and Welfare in Zipf's World\*

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## Abstract

Firm size follows Zipf's Law, a very fat-tailed distribution that implies a few large firms account for a disproportionate share of overall economic activity. This distribution of firm size is crucial for evaluating the welfare impact of macroeconomic policies such as barriers to entry or trade liberalization. Using a multi-country model of production and trade in which the parameters are calibrated to match the observed distribution of firm size, we show that the welfare impact of high entry costs is small. In the sample of the largest 50 economies in the world, a reduction in entry costs all the way to the U.S. level leads to an average increase in welfare of only 3.25%. In addition, when the firm size distribution follows Zipf's Law, the welfare impact of the extensive margin of trade – newly imported goods – vanishes. The extensive margin of imports accounts for only about 3.5% of the total gains from a 10% reduction in trade barriers in our model. This is because under Zipf's Law, the large, inframarginal firms have a far greater welfare impact than the much smaller firms that comprise the extensive margin in these policy experiments. The distribution of firm size matters for these results: in a counterfactual model economy that does not exhibit Zipf's Law the gains from a reduction in entry barriers are an order of magnitude larger, while the gains from trade liberalization are an order of magnitude smaller.

*JEL Classifications:* F12, F15

*Keywords:* Zipf's Law, welfare, entry costs, trade barriers

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# 1 Introduction

There is growing recognition that the structure of the economy is important for understanding economic outcomes as well as the impact of policies.<sup>1</sup> This paper investigates the implications of one particular characteristic of the economy – the distribution of firm size. The most striking fact about the firm size distribution is that it is extremely fat tailed. Several studies have documented that it follows a power law with an exponent close to  $-1$ , a result known as Zipf’s Law.<sup>2</sup>

The literature has sought to explore under what conditions Zipf’s Law can arise (see, e.g., Gabaix 1999, Luttmer 2007, Rossi-Hansberg and Wright 2007). By contrast, much less is known about how this phenomenon affects economic outcomes.<sup>3</sup> This paper explores how Zipf’s Law affects our conclusions about the welfare impact of two policy changes: a reduction in barriers to firm entry and trade opening. We first show that the welfare implications of these policies are very sensitive to the assumption regarding the firm size distribution. We then quantify their welfare impact in a calibrated multi-country general equilibrium model of production and trade, emphasizing the channels at work and the features of the economy that are crucial for the results.

Ever since the influential work of Djankov, La Porta, Lopez-De-Silanes and Shleifer (2002), it has been known that cross-country differences in the cost of entry by firms are pronounced. These authors assemble data on the entry regulations in 85 countries, and document that the amount of time, the number of procedures, and the costs – in either dollar terms or as a percentage of per capita income – required to start a business vary widely between countries.<sup>4</sup> The World Bank’s Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. Almost invariably, the data show that the variation in these regulations across countries is considerable. In addition, in a cross section of countries entry barriers are robustly negatively correlated with per-capita income and other measures of welfare.

Parallel to the research on entry barriers, the recent advances in international trade have focused attention on the role of individual firms, both in theory and the empirics. Naturally, when the unit of the analysis is the firm, much of the emphasis has been placed on the entry decision into export markets, the so-called “extensive margin.” Many stylized facts have emerged: most firms do not

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<sup>1</sup>To give but a few recent examples, Hsieh and Klenow (2008) demonstrate that correcting for misallocation of resources across firms in India and China can raise TFP in those countries by as much as 50%; Carvalho (2008) finds that aggregate fluctuations can arise from sectoral shocks given the observed structure of the Input-Output matrix; and Ghironi and Melitz (2005) show that the exporting behavior of individual firms helps explain the persistence of PPP deviations.

<sup>2</sup>Axtell (2001) provides empirical evidence that the distribution of sales and employment in the Census of U.S. firms follows Zipf’s Law. Similar findings obtain for several European countries (Fujiwara, Aoyama, Di Guilmi, Souma and Gallegati 2004) and Japan (Okuyama, Takayasu and Takayasu 1999). Other phenomena known to follow power laws include city size, income and wealth, and CEO compensation (Gabaix 2008).

<sup>3</sup>Exceptions include Gabaix (2009) and di Giovanni and Levchenko (2009), who study the implications of Zipf’s Law for macroeconomic volatility, and Gabaix and Landier (2007), who examine its relationship to executive compensation.

<sup>4</sup>To give one example, the official cost of following all the procedures to set up a business ranges from 0.5% of per capita GDP in the U.S. to 4.6 times per capita GDP in the Dominican Republic.

export, most exporters only sell small amounts abroad, while the bulk of exports at any one point in time is accounted for by a relatively small number of firms (see, e.g. Bernard, Jensen, Redding and Schott 2007). The extensive margin has been the focus of several theoretical and quantitative exercises, such as Ghironi and Melitz (2005), Chaney (2008), and Helpman, Melitz and Rubinstein (2008), among many others.

If entry is important – be it into production, or the export markets – it is becoming clear that one of the ways it matters is through the varieties available as intermediate inputs in production. Jones (2007, 2008) shows that the use of intermediate inputs creates a TFP multiplier that goes some way to explaining observed income differences across countries. Cowan and Neut (2007) were the first to argue that in countries with worse institutions, production will use fewer intermediates, adversely affecting productivity. Acemoglu, Antràs and Helpman (2007) and Costinot (2009) provide models that endogenize the number of varieties used in production as a function of economic institutions. On the trade side, it has also been argued that imported intermediates play an important role in domestic productivity.<sup>5</sup> This suggests that in order to assess the welfare impact of entry, our model must feature foreign and domestic intermediate inputs and the associated multiplier.

How does the existence of Zipf’s Law inform our conclusions about the importance of domestic or foreign entry, be it for consumption or as intermediate inputs? In other words, how much does the extensive margin matter in a world dominated by the very large firms? To answer this question, this paper sets up a workhorse multi-country model of international trade in the spirit of Melitz (2003) and Eaton, Kortum and Kramarz (2008), and calibrates it paying special attention to the parameters governing the size distribution of firms. In particular, we choose the model parameter values such that firm size follows Zipf’s Law. In addition, to capture the variation in fixed costs of entry across countries, our calibration uses entry cost data from the World Bank’s Doing Business Indicators database.

In a quantitative exercise, it is important to match the distribution of firm size because Zipf’s Law is a very special distribution. A random variable generating a power law with exponent between  $-1$  and  $-2$  has infinite variance. When the power law exponent is less than 1 in absolute value, the mean becomes infinite as well. Another way to put this is that the economy is dominated by a small number of very large firms, or to use a term coined by Gabaix (2009), the economy is “granular.” We adopt this terminology throughout the paper, and refer to the calibration using Zipf’s Law as the granular calibration. We then contrast the quantitative implications of Zipf’s Law with an exercise in which the distribution of firm size has finite variance (labeling it the “non-granular” case).

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<sup>5</sup>For instance, Amiti and Konings (2007), Kasahara and Rodrigue (2008), Luong (2008), and Halpern, Koren and Szeidl (2009) provide empirical evidence that newly available foreign intermediate inputs increased the TFP of individual firms.

The main results can be summarized as follows. First, welfare gains from a reduction in fixed costs of entry in the granular model are modest. In the sample of the largest 50 economies in the world, a reduction in entry costs all the way to the U.S. level leads to an average increase in welfare of only 3.25%. It turns out that the assumption of Zipf's Law matters a great deal here. Gains are about 12 times higher in the non-granular calibration compared to the granular one. Second, the gains from a 10% reduction in variable trade costs are far greater (by a factor of 15) in the granular model compared to the non-granular one. Third, the intensive margin, given by the reduction in the prices of existing imports, accounts for almost 98% of the total welfare impact of trade barrier reduction. Somewhat surprisingly, the relative importance of the intensive margin is not affected much by the Zipf's Law assumption. And finally, the extensive margin of foreign varieties accounts for only 3.6% of the gains from a reduction in trade barriers in the granular calibration, compared to 9% in the non-granular one.<sup>6</sup>

In summary, the distribution of firm size matters a great deal for whether fixed or variable costs have a larger welfare impact. In fact, depending on whether the firm size distribution is granular or not, the conclusions are reversed: in the granular world fixed costs matter little, while variable costs a great deal; the opposite is true in the non-granular world.

What is the intuition for these results? The distribution of firm size is informative about the relative importance of marginal exporters compared to the inframarginal ones for welfare. In a granular world, the marginal exporters are far less productive, and therefore much smaller and sell much less. As a result, their weight in the price index (this index corresponding roughly to the inverse of welfare) is extremely low. By contrast, the inframarginal, extremely large firms sell a lot and carry a large weight in the price index. Therefore, what happens to the large firms has a first-order impact on welfare. Put simply, suppose a country is already importing the most successful brands of television sets: Sony, Panasonic, etc. A reduction in variable trade costs makes these existing brands cheaper, but also has an impact on the extensive margin: it introduces many more inferior brands of televisions into the country. The model calibrated to the empirically observed distribution of firm size is telling us that the intensive margin – cheaper Sony TV sets – matters far more for welfare than the many additional bad brands that are now available. In fact, we show analytically that in the limit as the model parameters approach the values observed in the data, the welfare impact of the extensive margin of foreign trade goes to zero.

We are not the first to note that the welfare gains from new varieties may not be very large. Arkolakis, Demidova, Klenow and Rodriguez-Clare (2008) argue that the welfare gains from new imported goods will depend on the productivity of new varieties relative to existing ones, and that

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<sup>6</sup>The disappearing domestic varieties (the domestic extensive margin) have a correspondingly negative welfare impact.

in practice, the price level adjustment due to new varieties is likely to be small.<sup>7</sup> In addition, they show that in the standard model of monopolistic competition with endogenous variety, gains from trade are summarized by the overall trade volume relative to domestic absorption, something that is also true in our model. Feenstra (2009) argues that in a Melitz model with free entry, the positive welfare impact of newly imported varieties is exactly cancelled out by the negative welfare impact of disappearing domestic varieties, resulting in gains from variety that are precisely nil.<sup>8</sup>

Conceptually, our paper offers two innovations. First, we make it explicit that the observed distribution of firm size is informative about the relative importance of entry costs and the extensive margin of imports, and use it to discipline the quantitative exercise. We contrast the welfare impacts of fixed versus variable trade costs, and show how the distribution of firm size affects their relative magnitudes. Our results complement Feenstra’s by demonstrating that under Zipf’s Law, the welfare impact of not only the “net extensive margin” – foreign plus domestic – but also of the “gross extensive margin” – foreign and domestic individually – vanishes. In a sense, this is a stronger result as it does not depend on the two gross margins cancelling out perfectly. Instead we show that they are both vanishingly small in absolute value. And second, from a modeling standpoint, our analysis features foreign and domestic varieties as intermediate inputs in production. This turns out to be important: as we show below, the introduction of intermediate inputs affects both analytical and quantitative results. Finally, this paper develops the quantitative implications of reductions in entry costs and trade barriers for both the intensive and the extensive margins in a calibrated multi-country model.

Before moving on to the description of the model, a caveat is in order for interpreting the results. Our quantitative exercise does not strictly speaking tell us that the extensive margin does not matter for welfare. As such, it is not in direct contradiction with the empirical studies that find a welfare impact of increased varieties (Broda and Weinstein 2006, Goldberg, Khandelwal, Pavcnik and Topalova 2008). What our results demonstrate is that if the extensive margin is to matter for welfare, it would be through channels not captured by the standard model in this paper. This is important because the literature so far has overwhelmingly used this type model for the study of the extensive margin. In other words, some other mechanisms need to be specified for the extensive margin to have a discernible welfare impact.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. We show how the parameters of the model govern the distribution of firm size, and how they can be mapped into the empirical firm size distribution. We then derive a number of analytical results that foreshadow the conclusions from the quantitative exercise. Section 3 solves the model economy

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<sup>7</sup>In a dynamic two-country model of trade and innovation, Atkeson and Burstein (2008) argue that the impact of the extensive margin on the rate of innovation is likely to be small as well.

<sup>8</sup>This result does not appear to hold in a Chaney (2008)-type model with a fixed mass of entrepreneurs.

numerically and presents the main quantitative results. Section 4 concludes.

## 2 Theoretical Framework

The world is comprised of  $N$  countries, indexed by  $i, j = 1, \dots, N$ . In country  $i$ , buyers (who could be final consumers or firms buying intermediate inputs) maximize

$$\begin{aligned} \max \left[ \int_{J_i} Q_i(k) \frac{\varepsilon-1}{\varepsilon} dk \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ \text{s.t.} \\ \int_{J_i} p_i(k) Q_i(k) dk = X_i, \end{aligned}$$

where  $Q_i(k)$  is the quantity sold of good  $k$  in country  $i$ ,  $p_i(k)$  is the price of this good,  $X_i$  is total expenditure in the economy, and  $J_i$  is the number of varieties consumed in country  $i$ , coming from all countries. It is well known that demand for variety  $k$  is equal to

$$Q_i(k) = \frac{X_i}{P_i^{1-\varepsilon}} p_i(k)^{-\varepsilon} \quad (1)$$

in country  $i$ , where  $P_i$  is the ideal price index in this economy,

$$P_i = \left[ \int_{J_i} p_i(k)^{1-\varepsilon} dk \right]^{\frac{1}{1-\varepsilon}}. \quad (2)$$

Each country has a fixed number of potential (but not actual) entrepreneurs  $n_i$ , as in Eaton et al. (2008), Chaney (2008), and Arkolakis (2008). Each potential entrepreneur can produce a unique CES variety, and thus has some market power: it faces the demand for its variety given by (1). There are both fixed and variable costs of production and trade. Each entrepreneur's type is given by the marginal cost  $a(k)$ . On the basis of this cost, each entrepreneur in country  $i$  decides whether or not to pay the fixed cost of production  $f_{ii}$ , and which, if any, export markets to serve. To start exporting from country  $j$  to country  $i$ , a firm must pay the fixed cost  $f_{ij}$ , and an iceberg per-unit cost of  $\tau_{ij} > 1$ .<sup>9</sup>

There is one factor of production, labor, with country endowments given by  $L_j$ ,  $j = 1, \dots, N$ . Production uses both labor and intermediate inputs. In particular, the entrepreneur with marginal cost  $a(k)$  must use this many input bundles to produce one unit of output. An input bundle consists of labor and an aggregate of intermediate inputs, and has a cost  $c_j = w_j^\beta P_j^{1-\beta}$ , where  $w_j$  is the wage of workers in country  $j$ , and  $P_j$  is, as above, the ideal price index of all varieties available in  $j$ . Firm  $k$  from country  $j$  selling to country  $i$  faces a demand curve given by (1), and has a marginal cost  $\tau_{ij} c_j a(k)$  of serving this market. As is well known, the profit maximizing price is a constant markup

<sup>9</sup>That is, the firm in country  $j$  must ship  $\tau_{ij} > 1$  units to country  $i$  in order for one unit of the good to arrive there. We normalize the iceberg cost of domestic sales to one:  $\tau_{ii} = 1$ .

over marginal cost,  $p_i(k) = \frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k)$ , the quantity supplied is equal to  $\frac{X_i}{P_i^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k) \right)^{-\varepsilon}$ , and the total ex-post variable profits are:

$$\pi_{ij}^V(k) = \frac{X_i}{\varepsilon P_i^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k) \right)^{1-\varepsilon}. \quad (3)$$

Note that these are variable profits of a firm in country  $j$  from selling its good to country  $i$  only. These expressions are valid for each country pair  $i, j$ , including domestic sales:  $i = j$ .

The production structure of the economy is pinned down by the number of firms from each country that enter each market. In particular, there is a cutoff marginal cost  $a_{ij}$ , above which firms in country  $j$  do not serve market  $i$ . We assume (and later verify in the calibration exercise), that all firms that decide to export abroad are sufficiently productive to also serve their domestic markets. On the other hand, there is a range of productivities for which firms serve their domestic markets, but choose not to export. In this case, firms with marginal cost above  $a_{jj}$  in country  $j$  do not operate at all. The cutoff  $a_{ij}$  characterizes the entrepreneur in  $j$  who earns zero profits from shipping to country  $i$ :

$$a_{ij} = \frac{\varepsilon-1}{\varepsilon} \frac{P_i}{\tau_{ij} c_j} \left( \frac{X_i}{\varepsilon c_j f_{ij}} \right)^{\frac{1}{\varepsilon-1}}. \quad (4)$$

Closing the model involves finding expressions for  $a_{ij}$ ,  $P_i$ , and  $w_i$  for all  $i, j = 1, \dots, N$ . The price level for country  $i$  can be expressed as follows:

$$P_i = \left\{ \sum_{j=1}^N \int_{J_{ij}} \left[ \frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k) \right]^{1-\varepsilon} dk \right\}^{\frac{1}{1-\varepsilon}},$$

where  $J_{ij}$  is the set of varieties exported from country  $j$  to country  $i$ . In order to solve the model, we make the standard assumption that productivity,  $1/a$ , is Pareto( $b, \theta$ ), where  $b$  is the minimum value productivity can take, and  $\theta$  regulates dispersion. The cdf of productivity is given by:

$$\Pr(1/a < x) = 1 - \left( \frac{b}{x} \right)^\theta.$$

It is then straightforward to show that the marginal cost,  $a$ , has a distribution function  $G(a) = (ba)^\theta$ . The price level then becomes, after plugging in the expression for  $a_{ij}$  in (4):

$$P_i = \left( \sum_{j=1}^N n_j \int_0^{a_{ij}} \left[ \frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a \right]^{1-\varepsilon} dG(a) \right)^{\frac{1}{1-\varepsilon}} \quad (5)$$

$$= \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{X_i}{\varepsilon} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \left( \sum_{j=1}^N n_j \left( \frac{1}{\tau_{ij} c_j} \right)^\theta \left( \frac{1}{c_j f_{ij}} \right)^{\frac{\theta - (\varepsilon - 1)}{\varepsilon - 1}} \right)^{-\frac{1}{\theta}}. \quad (6)$$

Having expressed  $P_i$  and  $a_{ij}$  in terms of  $X_i$  and  $c_i$ , for all  $i, j = 1, \dots, N$ , it remains to close the model by solving for the  $X_i$ 's and  $w_i$ 's. To do this, we impose balanced trade for each country, and



use the convenient property (originally noted by Eaton and Kortum 2005) that total profits in the economy are a constant multiple of  $X_i$ .

**Proposition 1** *Total profits of firms based in country  $i$  are a constant multiple of total expenditure:  $\Pi_i = \frac{\varepsilon-1}{\theta\varepsilon} X_i$ .*

**Proof:** See Appendix A. ■

Since by definition total sales in the economy are equal to  $X_i$ , and the total profits are  $\frac{\varepsilon-1}{\theta\varepsilon} X_i$ , the total spending on inputs is  $(1 - \frac{\varepsilon-1}{\theta\varepsilon}) X_i$ . Labor receives a constant fraction  $\beta$  of the spending on inputs. Thus, each country's GDP is a constant multiple its total labor income:

$$X_i = \frac{1}{\beta \left(1 - \frac{\varepsilon-1}{\theta\varepsilon}\right)} w_i L_i. \quad (7)$$

The total sales from country  $i$  to country  $j$  can be written as:

$$X_{ji} = \frac{X_j}{P_j^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ji} c_i \right)^{1-\varepsilon} n_i \frac{b^\theta \theta}{\theta - (\varepsilon-1)} a_{ji}^{\theta - (\varepsilon-1)}.$$

Using the expression for  $a_{ji}$  in (4), and  $P_j$  in (6), total exports from  $i$  to  $j$  become:

$$X_{ji} = \frac{n_i \left( \frac{1}{\tau_{ji} c_i} \right)^\theta \left( \frac{1}{c_i f_{ji}} \right)^{\frac{\theta - (\varepsilon-1)}{\varepsilon-1}}}{\sum_{l=1}^N n_l \left( \frac{1}{\tau_{jl} c_l} \right)^\theta \left( \frac{1}{c_l f_{jl}} \right)^{\frac{\theta - (\varepsilon-1)}{\varepsilon-1}}} X_j. \quad (8)$$

Using the trade balance conditions,  $X_i = \sum_{j=1}^N X_{ji}$  for each  $i = 1, \dots, N$ , the expression for total GDP,  $X_i$ , in (7), and the definition of  $c_i$  leads to the following system of equations in  $w_i$ :

$$w_i L_i = \sum_{j=1}^N \frac{n_i \left( \frac{1}{\tau_{ji} w_i^\beta P_i^{1-\beta}} \right)^\theta \left( \frac{1}{w_i^\beta P_i^{1-\beta} f_{ji}} \right)^{\frac{\theta - (\varepsilon-1)}{\varepsilon-1}}}{\sum_{l=1}^N n_l \left( \frac{1}{\tau_{jl} w_l^\beta P_l^{1-\beta}} \right)^\theta \left( \frac{1}{w_l^\beta P_l^{1-\beta} f_{jl}} \right)^{\frac{\theta - (\varepsilon-1)}{\varepsilon-1}}} w_j L_j, \quad (9)$$

$i = 1, \dots, N$ . There are  $N - 1$  independent equations in this system, which can be solved for wages in  $N - 1$  countries given a numéraire wage in the remaining country. The wages and the price levels in all countries are determined jointly by equations (9) for wages and (6) for prices. We will solve these numerically in order to carry out the main quantitative exercise in this paper.

## 2.1 The Distribution of Firm Size: Model and Data

It has been argued that in the data, the distribution of firm size follows a power law, with an exponent close to 1 in absolute value. In this section, we first build a bridge between the model and the data by showing that the distribution of firm sales in the workhorse model outlined above

does indeed follow a power law. Consequently, we argue that the distribution of firm size in the data places a key restriction on the important parameter values in the model. Finally, we review the available empirical evidence on the firm size distribution.

Denote by lower case  $x(a(k))$  the sales of an individual firm  $k$ . Firm sales  $x$  follow a power law if

$$\Pr(x > s) = cs^{-\zeta}. \quad (10)$$

It turns out that the baseline Melitz-Pareto model delivers a power law in firm size. In our model, the sales of a firm as a function of its marginal cost are:  $x(a) = Ca^{1-\varepsilon}$ , where the constant  $C$  reflects the size of overall demand, and we drop the country subscripts. Under the assumption that  $1/a \sim \text{Pareto}(b, \theta)$ , the power law follows:

$$\begin{aligned} \Pr(x > s) &= \Pr(Ca^{1-\varepsilon} > s) = \Pr\left(a^{1-\varepsilon} > \frac{s}{C}\right) = \\ \Pr\left(\left(\frac{1}{a}\right)^{\varepsilon-1} > \frac{s}{C}\right) &= \Pr\left(\frac{1}{a} > \left(\frac{s}{C}\right)^{\frac{1}{\varepsilon-1}}\right) = \left(\frac{b^{\varepsilon-1}C}{s}\right)^{\frac{\theta}{\varepsilon-1}} = (b^{\varepsilon-1}C)^{\frac{\theta}{\varepsilon-1}} s^{-\frac{\theta}{\varepsilon-1}} \end{aligned}$$

satisfying (10) for  $c = (b^{\varepsilon-1}C)^{\frac{\theta}{\varepsilon-1}}$  and  $\zeta = \frac{\theta}{\varepsilon-1}$ . In addition, this calculation shows that  $x \sim \text{Pareto}\left(b^{\varepsilon-1}C, \frac{\theta}{\varepsilon-1}\right)$ .

The key point for connecting the model to the data is that in the model, the slope of the power law is given by  $\frac{\theta}{\varepsilon-1}$ . Since this exponent can also be estimated in the data, what we observe in the data is informative about this combination of parameters. What do the data tell us about  $\zeta$ ? Available estimates put it very close to 1, suggesting that the distribution of firm size follows Zipf's Law. Figure 1 reproduces the now famous power law for firm size in the U.S. estimated by Axtell (2001). The fit of this relationship is typically very close: it is common to observe R-squareds in excess of 0.99. Using a variety of estimation techniques, Axtell reports a range of estimates of  $\zeta$  between 0.996 and 1.059, very precisely estimated with a standard errors between 0.054 and 0.064. It will become important below that the coefficient estimates are never significantly different from 1, and indeed never very far from 1 in absolute terms as well.

The question remains whether Zipf's Law obtains in the firm size distributions for many countries. Currently, no comprehensive set of results exists. Evidence for a limited set of European countries is presented by Fujiwara et al. (2004) and for Japan by Okuyama et al. (1999). In Appendix C, we use ORBIS/AMADEUS – the largest publicly available firm-level dataset covering a large number of countries – to show that firm size distributions are well approximated by a power law, with exponents quite close to  $-1$  in most countries.<sup>10</sup>

<sup>10</sup>Other related results also shed light on how fat-tailed size distributions are. For instance, it turns out that measures of Balassa revealed comparative advantage also follow power laws with exponent close to  $-1$  (Hinloopen and van Marrewijk 2006)

To summarize, existing estimates of the distribution of firm size put discipline on the parameters of the Melitz-Pareto model. In particular, estimates suggest that  $\frac{\theta}{\varepsilon-1}$  is very close to 1. As we show in a series of exercises below, this has some striking implications regarding gains from reductions in entry barriers and trade costs, the relative importance of intensive and extensive margins, and the ability of trade openness to explain income differences between countries.

## 2.2 Entry Costs, Trade Openness, and the Magnitude of Gains from Trade

We now present a number analytical results about the relative importance of fixed costs, trade openness, and the extensive margin for welfare. Real income per capita in country  $i$  is proportional to  $w_i/P_i$ , which is also a measure of welfare.<sup>11</sup> It is possible to use trade shares to simplify the expression for the price level. Define  $\pi_{ij} \equiv X_{ij}/X_i$  to be the share of total spending in country  $i$  on goods from country  $j$ . Using equation (8), setting  $i = j$  and rearranging yields the following relationship:

$$\sum_{l=1}^N n_l \left( \frac{1}{\tau_{il}c_l} \right)^\theta \left( \frac{1}{c_l f_{il}} \right)^{\frac{\theta-(\varepsilon-1)}{\varepsilon-1}} = \frac{1}{\pi_{ii}} n_i \left( \frac{1}{c_i} \right)^\theta \left( \frac{1}{c_i f_{ii}} \right)^{\frac{\theta-(\varepsilon-1)}{\varepsilon-1}}.$$

Plugging this expression into the price level (6) and rearranging, welfare under trade in this economy can be written as:

$$\frac{w_i}{P_i} = \left\{ \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} n_i^{\frac{\theta}{\theta}} \left( \frac{L_i}{f_i \varepsilon \beta \left( 1 - \frac{\varepsilon-1}{\varepsilon \theta} \right)} \right)^{-\frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}} \pi_{ii}^{\frac{1}{\theta}} \right\}^{-\frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}}. \quad (11)$$

This allows us to represent real income per capita in each country relative to the U.S. as a product of several components:

$$\begin{aligned} \frac{w_i/P_i}{w_{US}/P_{US}} &= \left( \frac{n_i}{n_{US}} \right)^{\frac{1}{\theta} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}} \left( \frac{L_i}{L_{US}} \right)^{\frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}} \times \\ &\quad \left( \frac{f_{ii}}{f_{US,US}} \right)^{-\frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}} \left( \frac{\pi_{ii}}{\pi_{US,US}} \right)^{-\frac{1}{\theta} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}}. \end{aligned}$$

A special case of this expression is obtained if we adopt the assumption in Alvarez and Lucas (2007), Chaney (2008), and di Giovanni and Levchenko (2009) that the number of productivity draws in each country is proportional to its size:  $n_i = \gamma L_i$ , where  $\gamma$  is a constant. In that case, income differences can be decomposed as:

$$\frac{w_i/P_i}{w_{US}/P_{US}} = \left( \frac{L_i}{L_{US}} \right)^{\frac{1}{(\varepsilon-1)} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}} \left( \frac{f_{ii}}{f_{US,US}} \right)^{-\frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}} \left( \frac{\pi_{ii}}{\pi_{US,US}} \right)^{-\frac{1}{\theta} \frac{1}{\beta - (1-\beta) \frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}}}.$$

<sup>11</sup>Welfare is proportional to the real wage even though in this economy there are profits. From Proposition 1, profits are a constant multiple of the total expenditure, while due to the Cobb-Douglas functional form of the input bundle, the wage bill  $w_i L_i$  is a constant multiple of total expenditure as well. Hence, the total profits in the economy are a constant multiple of the wage bill, making the total welfare proportional to the real wage. See eq. (7).

This expression is similar in spirit to Waugh (2007), with some key differences. The similarity is in the contribution of trade to income differences, which is summarized simply by the relative openness ( $\frac{\pi_{ii}}{\pi_{US,US}}$ ). The difference is that in our model entry costs also matter (the  $\frac{f_{ii}}{f_{US,US}}$  term), and there is a “home market effect,” such that larger countries have lower price levels and higher real per-capita incomes, all else equal.

We can get a sense of the magnitudes involved by examining both the variation in the relative fixed costs and openness, as well as the exponents. We choose the parameter values as follows:  $\beta = 0.5$  from Jones (2008),  $\varepsilon = 6$  (Anderson and van Wincoop 2004), and  $\theta = 5.3$ , designed to match the power law exponent on firm size to U.S. data,  $\frac{\theta}{\varepsilon-1} = 1.06$  (Axtell 2001). Then, the exponents in the expression above become:

$$\frac{w_i/P_i}{w_{US}/P_{US}} = \left(\frac{L_i}{L_{US}}\right)^{0.40} \left(\frac{f_{ii}}{f_{US,US}}\right)^{-0.02} \left(\frac{\pi_{ii}}{\pi_{US,US}}\right)^{-0.38}.$$

It is immediate that the relative fixed costs will matter far less than the other two terms. In a granular economy, what is really important for welfare is the presence of the large, very productive firms, which are inframarginal and not affected much by the level of fixed costs.

To make this more precise, we use the World Bank’s Doing Business Indicators to measure variation in  $\frac{f_{ii}}{f_{US,US}}$  present in the data, and compute how much per-capita income variation those can generate. It turns out that the country at the 95th percentile of the fixed cost distribution has an  $f_{ii}$  that is between 16 and 658 times the U.S. value, depending on the precise indicator we use. Plugging those ratios into the equation above, we get that the country at the 95th percentile of fixed entry costs has an income level between 0.86 and 0.94 that of the U.S., all else equal. In a granular economy, differences in fixed costs of entry cannot generate large per-capita income – and welfare – differences.

What about trade? In the sample of the 49 largest economies by total GDP, the ratio  $\frac{\pi_{ii}}{\pi_{US,US}}$  for the economy in the 95th percentile of openness is 0.577. Taking that to the correct exponent implies that this country has an income level 1.23 times that of the U.S. While the absolute variation in  $\frac{\pi_{ii}}{\pi_{US,US}}$  in the data is far lower than the variation in fixed costs, the impact of trade openness on welfare is larger.

The distribution of firm size matters for these magnitudes. To see what happens when we depart from Zipf’s Law, we set  $\frac{\theta}{\varepsilon-1}$  equal to 2 (implying a value of  $\theta = 10$  given our chosen elasticity of substitution). When the exponent on the power law in firm size is greater than or equal to 2, the distribution of firm size has finite variance, and thus in Gabaix (2009)’s terminology, such an economy is no longer granular. Thus, in our non-granular calibrations we set the exponent on the power law in firm size to be the smallest such that the distribution still has a finite variance.

In a non-granular economy, the exponents change dramatically: on the  $\frac{f_{ii}}{f_{US,US}}$  term, the ex-

ponent goes up from 0.02 to 0.22 in absolute value, a tenfold increase. By contrast, the exponent on the  $\frac{\pi_{ii}}{\pi_{US,US}}$  term drops by almost half, from 0.38 to 0.22. This implies that the importance of fixed costs rises: now, a country in the 95th percentile of the  $f_{ii}$  distribution has the income level between 0.23 and 0.54 that of the U.S.. By contrast, the contribution of trade drops by half: the 95th percentile most open country has income per capita only about 1.12 of the U.S. level.

As a related point, granularity matters a great deal for the magnitude of gains from trade. In this model, gains from trade are equal to:<sup>12</sup>

$$\pi_{ii}^{-\frac{1}{\theta} \frac{1}{\beta - (1-\beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}}} \quad (12)$$

A few things are notable about this expression. First, at a given  $\pi_{ii}$ , gains from trade are decreasing in  $\frac{\theta}{\varepsilon - 1}$  as long as  $\beta\varepsilon > 1$ .<sup>13</sup> In other words, the more granular is the economy, the larger are the gains from trade. This is intuitive: in the world dominated by ultra-productive firms, the big gains from trade come from having access to those extremely productive foreign varieties. Using the values of  $\beta$ ,  $\varepsilon$ , and  $\theta$  described above, in the sample of 50 largest economies in the world, average gains from trade are 13%, with a standard deviation of 11% across countries. Assuming instead that  $\frac{\theta}{\varepsilon - 1} = 2$  (the economy is not granular) reduces the estimated mean gains almost in half (7%), and the variation across countries in half as well (standard deviation of 6%).

Second, at a given level of trade openness ( $\pi_{ii}$ ), gains from trade are increasing in the share of intermediate goods in the input bundle,  $(1 - \beta)$ . This is an intermediate goods multiplier effect akin to Jones (2008): the more foreign varieties are used as intermediate goods in production, the more the country reaps a double benefit from trade: first as an increase in labor productivity due to foreign intermediates, and second as consumers of those foreign varieties.

Finally, in order to get a sense of the gains from trade, it is sufficient to simply look at the share of spending on domestic goods. This feature has been noted about Ricardian models (Eaton and

<sup>12</sup>To find an expression for gains from trade, it is useful to write out the autarky price level and welfare. Setting  $N = 1$  in equation (6) and dropping country subscripts, the autarky price level becomes:

$$P_A = \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} n^{-\frac{1}{\theta}} c \left( \frac{X}{\varepsilon c f} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}}.$$

Using the expression for  $X$  in (7) and  $c$ , we can write welfare in autarky as follows:

$$\frac{w_A}{P_A} = \left\{ \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} n^{-\frac{1}{\theta}} \left( \frac{L}{f\varepsilon\beta \left(1 - \frac{\varepsilon - 1}{\varepsilon\theta}\right)} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \right\}^{-\frac{1}{\beta - (1-\beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}}}.$$

It is immediate from comparing this expression for autarky welfare to (11) that the two differ only by the term  $\pi_{ii}^{-\frac{1}{\theta} \frac{1}{\beta - (1-\beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}}}$ , yielding equation (12).

<sup>13</sup>This latter condition is likely to be satisfied in the data. Typical estimates of  $\varepsilon$  range from 3 to 10, while  $\beta$  is on the order of 0.5 (Jones 2008).

Kortum 2002), as well as monopolistic competition models such as the one in this paper (Arkolakis et al. 2008).

At the same time, in the presence of intermediate goods it is no longer the case that in order to calculate the overall gains from trade, one needs to know only  $\pi_{ii}$  and the estimated elasticity of trade with respect to (variable) trade costs, as argued by Arkolakis et al. (2008). As can be gleaned from equation (8), the elasticity of bilateral trade with respect to  $\tau_{ji}$  is  $-\theta$ . It is true that, without intermediate goods, the gains from trade are given by  $\pi_{ii}^{-\frac{1}{\theta}}$ , so that the exponent on  $\pi_{ii}$  is exactly the inverse of that elasticity. However, in the presence of intermediate inputs, that is not the case: the exponent on  $\pi_{ii}$  is now  $-\frac{1}{\theta} \frac{1}{\beta - (1-\beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}}$ . In other words, in order to assess the gains from trade, we can no longer rely on one potentially observable object – the elasticity of bilateral trade with respect to  $\tau_{ji}$  – and instead need to take a stand on other parameters of the model, namely  $\beta$  and  $\varepsilon$ .

In this context, it is worth noting that the firm size distribution provides an alternative source of information regarding the model parameters. While at first blush one may believe that it is easy to estimate the elasticity of trade volumes with respect to trade costs, in fact one must typically make a series of parametric assumptions about how the true trade costs  $\tau_{ji}$  are related to observables such as distance or tariff barriers. Things are further complicated by the fact that in the Melitz model both  $\tau_{ji}$  and  $f_{ji}$  affect trade volumes, but with different elasticities. Since a typical gravity regression cannot distinguish between the two, yet more assumptions on the nature of fixed and variable costs are needed to back out  $\theta$ . The firm size distribution, by providing an estimate of  $\theta/(\varepsilon - 1)$ , is an arguably cleaner way to calibrate the parameters of the model. In fact, as we show in this paper, some key results actually depend on this combination of parameters, rather than  $\theta$  and  $\varepsilon$  individually.

### 2.3 Extensive vs. Intensive Margins

The granular economy is one dominated by few large producers, that are not likely to be “marginal” exporters. Intuitively, this suggests that the distribution of firm size will also affect the relative importance of intensive versus extensive margins for welfare. In this subsection we examine analytically the importance of the two margins. The conclusion is striking: as the firm-size distribution converges to Zipf’s Law, the welfare impact of extensive margin in exports (or indeed domestic production) goes to zero.

Going back to the definition of the price level, (5), write it as a function of the extensive margin as follows:

$$P_i = \left( \frac{\varepsilon}{\varepsilon - 1} b^{\varepsilon - 1} \frac{\theta}{\theta - (\varepsilon - 1)} \sum_{j=1}^N n_j (\tau_{ij} c_j)^{1 - \varepsilon} G(a_{ij})^{\frac{\theta - (\varepsilon - 1)}{\theta}} \right)^{\frac{1}{1 - \varepsilon}}. \quad (13)$$

Here, the price level is expressed in terms of the share of firms from country  $j$  supplying country  $i$ ,  $G(a_{ij})$ , precisely because it is the extensive margin: in any policy experiment, the change in  $G(a_{ij})$  is exactly the increase in the number (mass) of firms supplying market  $i$ . To derive the analytical result in the simplest way, let us assume that the countries are symmetric:  $L_i = L$ ,  $n_i = n$ ,  $f_{ii} = f \forall i$ , and  $\tau_{ij} = \tau$ ,  $f_{ij} = f_X \forall i, j, j \neq i$ . In that case, wages are the same in all countries, and we normalize them to 1. The price levels are the same in all countries as well, and thus dropping the country subscripts we obtain:

$$P = \left\{ \frac{\varepsilon}{\varepsilon - 1} b^{\varepsilon - 1} \frac{\theta}{\theta - (\varepsilon - 1)} n \left( G(a_D)^{\frac{\theta - (\varepsilon - 1)}{\theta}} + (N - 1) \tau^{1 - \varepsilon} G(a_X)^{\frac{\theta - (\varepsilon - 1)}{\theta}} \right) \right\}^{\frac{1}{\beta(1 - \varepsilon)}}, \quad (14)$$

where  $a_D$  is the cutoff for domestic production, and  $a_X$  is the cutoff for exporting. These are of course the same across all countries as well.

Note that since wages are normalized to 1, the total welfare in this economy is simply  $W = 1/P$ . We are now ready to evaluate the relative importance of the extensive and intensive margins. Imagine that there is a reduction in trade costs  $\tau$ . This reduction will affect both the prices that existing exporters charge in the domestic market, given by  $p(k) = \frac{\varepsilon}{\varepsilon - 1} \tau c a(k)$ , and the mass of firms serving the market,  $G(a_X)$ . From the expression for the price level (14), it is immediate that the elasticity of welfare with respect to the extensive margin is equal to:

$$\frac{d \log W}{d \log G(a_X)} = \frac{1}{\beta} \frac{\theta - (\varepsilon - 1)}{\theta} \frac{(N - 1) \tau^{1 - \varepsilon} G(a_X)^{\frac{\theta - (\varepsilon - 1)}{\theta}}}{G(a_D)^{\frac{\theta - (\varepsilon - 1)}{\theta}} + (N - 1) \tau^{1 - \varepsilon} G(a_X)^{\frac{\theta - (\varepsilon - 1)}{\theta}}}.$$

As the economy becomes granular –  $\theta \rightarrow (\varepsilon - 1)$  – the welfare impact of the extensive margin goes to zero:  $\frac{d \log W}{d \log G(a_X)} \rightarrow 0$ .

The same is not true for the intensive margin. The price  $p$  that each exporter charges in the domestic market is proportional to  $\tau$ . Therefore, the elasticity of welfare with respect to the intensive margin equals:

$$\frac{d \log W}{d \log p} = \frac{1}{\beta} \frac{(N - 1) \tau^{1 - \varepsilon} G(a_X)^{\frac{\theta - (\varepsilon - 1)}{\theta}}}{G(a_D)^{\frac{\theta - (\varepsilon - 1)}{\theta}} + (N - 1) \tau^{1 - \varepsilon} G(a_X)^{\frac{\theta - (\varepsilon - 1)}{\theta}}}.$$

The welfare impact of the intensive margin clearly does not converge to zero as  $\theta \rightarrow (\varepsilon - 1)$ .

What is the intuition for these results? In a granular economy the most productive firms are vastly better than the marginal firms. As a result, most of the welfare impact of trade is driven by what happens to these best firms, rather than by whether trade liberalization leads to new entry. That is, a reduction in trade costs impacts welfare mainly because the “major brands” – Sony, Panasonic, etc. – become cheaper, rather than because the many additional inferior brands of television sets become available.

This discussion shows that the conclusions about the impact of entry barriers, international trade, and the extensive margin are very sensitive to whether or not we think the economy is

granular. All else equal, the extensive margin matters less in the granular economy, and trade openness matters more, as it allows the country to access the extremely productive varieties from abroad.

Before proceeding to the quantitative assessment of the importance of entry costs and the intensive and extensive margins of trade in a multi-country calibrated model, it is worth making an additional remark regarding our modeling approach to fixed costs. Arkolakis (2008) develops a framework in which the fixed costs of entry are replaced by smoother market penetration costs, and firms choose not just whether to enter markets, but also what share of consumers to serve in each market. Appendix B presents a model with market penetration costs, and shows that proportional changes in welfare obtained in that model are identical to those in a simple fixed costs model of the main text. This result holds for all parameter values that govern the distribution of firm size and the curvature of market penetration costs. In addition, we show that as the distribution of firm size converges to Zipf’s Law, the *level* of welfare in that model also becomes identical to the baseline model. This is because under Zipf’s Law, what matters most for welfare are the very large firms, which are least affected by the introduction of the market penetration margin. The large firms choose to penetrate markets fully, making their sales nearly the same as what they would be in a simple fixed cost model. For this reason, we choose to adopt the standard formulation of fixed costs of entry in our analysis.

### 3 Quantitative Evidence

In order to fully solve the model numerically, we must find the wages and price levels for each country,  $w_i$  and  $P_i$ , using the system of equations given by (6) and (9). To solve this system, we must calibrate the values of  $L_i$ ,  $n_i$ ,  $\tau_{ij}$ , and  $f_{ij}$  for each country and country pair, as well as the parameters common to all countries. We now discuss how we calibrate each parameter value.

The elasticity of substitution is  $\varepsilon = 6$ . Anderson and van Wincoop (2004) report available estimates of this elasticity to be in the range of 3 to 10, and we pick a value close to the middle of the range. The key parameter is  $\theta$ , as it governs the slope of the power law. As described above, in this model firm sales follow a power law with the exponent equal to  $\frac{\theta}{\varepsilon-1}$ . In the data, firm sales follow a power law with the exponent close to 1. Axtell (2001) reports the value of 1.06, which we use to find  $\theta$  given our preferred value of  $\varepsilon$ :  $\theta = 1.06 \times (\varepsilon - 1) = 5.3$ . As mentioned above, we set the share of intermediates  $\beta = 0.5$ , following Jones (2008).

For finding the values of  $L_i$ , we follow the approach of Alvarez and Lucas (2007). First, we would like to think of  $L$  not as population per se, but as “equipped labor,” to take explicit account of TFP and capital endowment differences between countries. To obtain the values of  $L$  that are internally consistent in the model, we start with an initial guess for  $L_i$  for all  $i = 1, \dots, N$ , and use



it to solve the model. Given the vector of equilibrium wages, we update our guess for  $L_i$  for each country in order to match the ratio of total GDPs between each country  $i$  and the U.S.. Using the resulting values of  $L_i$ , we solve for the new set of wages, and iterate to convergence (for more on this approach, see Alvarez and Lucas 2007). Thus, our procedure generates vectors  $w_i$  and  $L_i$  in such a way as to match exactly the relative total GDPs of the countries in the sample. In practice, the results are extremely close to simply equating  $L_i$  to the relative GDPs of the countries. In this procedure, we must normalize the population of one of the countries. We thus set  $L_{US}$  to its actual value of 291 million as of 2003, and compute  $L_i$  of every other country relative to this U.S. value. Finally, we set  $n_i$  in proportion to  $L_i$ . That is, the country’s endowment of entrepreneurs is simply proportional to its “equipped labor” endowment. An important consequence of this assumption is that countries with higher TFP and capital abundance will have a greater number of potential productivity draws, all else equal. This is an assumption adopted by Alvarez and Lucas (2007) and Chaney (2008). We set  $n_{US} = 10,000,000$ , that is, there are ten million potential firms in the U.S.. In this calibration it implies that there are about 9,500,000 operating firms there. According to the 2002 U.S. Economic Census, there were 6,773,632 establishments with a payroll in the United States. There are an additional 17,646,062 business entities that are not employers, but they account for less than 3.5% of total shipments. Thus, choosing  $n_{US} = 10,000,000$  gets the correct order of magnitude for the number of firms.

Next, we must calibrate the values of  $\tau_{ij}$  for each pair of countries. To do that we use the set of gravity estimates from the empirical model of Helpman et al. (2008). That is, we combine geographical characteristics such as bilateral distance, common border, common language, whether the two countries are in a currency union and others, with the coefficient estimates reported by Helpman et al. (2008) to calculate values of  $\tau_{ij}$  for each country pair.<sup>14</sup> Note that in this formulation,  $\tau_{ij} = \tau_{ji}$  for all  $i$  and  $j$ .

Finally, we must take a stand on the values of  $f_{ii}$  and  $f_{ij}$ . The level of  $f_{US,US}$  is set to ensure an interior solution for the domestic production cutoff. Then, we use the information from the Doing Business Indicators database (The World Bank 2007a) to set  $f_{ii}$  for every other country relative to the U.S.. In this application, the particular Doing Business indicator is the amount of time required to set up a business. We favor this indicator compared to others that measure entry costs either in dollars or in units of per capita income, because in our model  $f_{ii}$  is a quantity of inputs rather than value. To be precise, if according to the Doing Business Indicators database, in country  $i$  it takes 10 times longer to register a business than in the U.S., then  $f_{ii} = 10 \times f_{US,US}$ .

To measure the fixed costs of international trade, we use the Trading Across Borders module

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<sup>14</sup>In di Giovanni and Levchenko (2009), as a robustness check, we also computed  $\tau_{ij}$  using the estimates of Eaton and Kortum (2002). The advantage of the Helpman et al. (2008) estimates is that they are obtained in an empirical model that accounts explicitly for both fixed and variable costs of exporting, and thus corresponds most closely to the theoretical structure in our paper.

of the Doing Business Indicators. This module provides the costs of exporting a 20-foot dry-cargo container out of each country, as well as the costs of importing the same kind of container into each country. We take the bilateral fixed cost  $f_{ij}$  to be the sum of the two: the cost of exporting from country  $j$  plus the cost of importing into country  $i$ .<sup>15</sup>

We carry out the analysis on the sample of the largest 49 countries by total GDP, plus the 50th that represents the rest of the world.<sup>16</sup> These 49 countries together cover 97% of world GDP. We exclude entrepôt economies of Hong Kong and Singapore, both of which have total trade well in excess of their GDP, due to significant re-exporting activity. Thus, our model is not intended to fit these countries. (We do place them into the rest-of-the-world category). The country sample, sorted by total GDP, is reported in Table 1.

### 3.1 Model Fit

As described above, our iterative procedure ensures that the ratio of total GDPs in the model for any two countries matches exactly the ratio of the total GDPs in the data. However, since the object of the paper is to examine the role of trade openness in welfare, it is more important that the model matches well the bilateral and overall trade volumes observed in the data. Comparing bilateral trade patterns generated by the model to the actual data is a good test of the model's success in describing the world economy, since the calibration procedure does not use any information on actual trade patterns, only country GDPs and estimated bilateral trade costs.

Figure 2 reports the scatterplot of bilateral trade ratios  $\pi_{ij} = X_{ij}/X_i$ . On the horizontal axis is the natural log of  $\pi_{ij}$  that comes from the model, while on the vertical axis is the corresponding value of that bilateral trade flow in the data. Hollow dots represent exports from one country to another,  $\pi_{ij}$ ,  $i \neq j$ . Solid dots, at the top of the scatterplot, represent sales of domestic firms as a share of domestic absorption,  $\pi_{ii}$ . For convenience, we added a 45-degree line. It is clear that the trade volumes implied by the model match the actual data well. Most observations are quite close to the 45-degree line. It is especially important that we get the overall trade openness ( $1 - \pi_{ii}$ ) right, since that will drive the gains from trade in each country. Figure 3 plots the actual values of ( $1 - \pi_{ii}$ ) against those implied by the model, along with a 45-degree line. We can see that though the relationship is not perfect, it is close.

Table 2 compares the means and medians of  $\pi_{ii}$  and  $\pi_{ij}$ 's for the model and the data, and reports the correlations between the two. The correlation between domestic shares  $\pi_{ii}$  the model and the data for this sample of countries is around 0.48. The means and the medians look very

<sup>15</sup>An earlier version of the paper carried out the analysis setting the bilateral fixed cost to be the sum of domestic costs of starting a business in the source and destination countries:  $f_{ij} = f_{ii} + f_{jj}$ . This approach may be preferred if fixed costs of exporting involved more than just shipping, and required, for instance, the exporting firm to create a subsidiary for the distribution in the destination country. The results were virtually identical.

<sup>16</sup>We set the parameters, such as  $\tau_{ij}$  and  $f_{ij}$ , for the rest-of-the-world category as the average values among the remaining countries in the world.

similar as well, with the countries in the model slightly more open on average than the data. The correlation between export shares,  $\pi_{ij}$ , is actually higher at 0.71.<sup>17</sup>

Overall, though the model calibration does not use any information on trade volumes, it fits bilateral trade data quite well. We now turn to the analysis of welfare gains from reduction in entry costs and trade barriers implied by the model.

### 3.2 Counterfactual I: Reduction in Entry Costs

Using the calibrated model above, the first counterfactual we perform is a reduction in the fixed costs of entry  $f_{ii}$  and  $f_{ij}$ . We simulate a complete harmonization of entry costs across the world, such that entry costs everywhere are the same as in the U.S.. This is a substantial improvement. As first shown by Djankov et al. (2002), the differences in these fixed costs are substantial across countries. In our sample of the world’s 49 largest economies, it takes on average 6 times longer to start a business compared to the U.S.. For a country at the 75th percentile of the distribution, it takes almost 8 times longer, and the country with the highest entry costs in this sample – Brazil – it takes 25 times longer than in the U.S.. This experiment also entails a substantial drop in the fixed costs of cross-border trade. The average exporting cost in this sample is 3 times higher than in the U.S., and the average importing cost is 4 times higher.

Table 3 reports the associated welfare gains. The top panel presents the granular calibration, in which firm-size distribution is set to match Zipf’s Law. The bottom panel reports the non-granular calibration, in which  $\theta/(\varepsilon - 1) = 2$ . Since by construction  $f_{ij}$  affects entry, but not the variable costs of existing firms, we attribute all of the welfare gains to the extensive margin. The welfare gains are small. We can see that even a dramatic drop (6-fold on average) in the fixed costs of production and exporting improves welfare by only 3.26% on average. It could be that this average number is hiding a lot of heterogeneity, since different countries are experiencing a different size reduction in trade costs. In parentheses below the average value, we report the range of welfare gains in the entire sample. We can see that even in the country that gains the most from this institutional improvement, the gain is only about double the average, at 7.32%. Zipf’s Law matters a great deal for this conclusion. In the bottom panel, we report that the welfare gain from the same reduction in entry barriers is on average 40.87% in the non-granular world. This is 12 times higher than in the Zipf’s Law calibration. The range is also greater: the country gaining the most more than doubles its welfare.<sup>18</sup>

<sup>17</sup>We also experimented with increasing the number of countries in the simulation to 60. The model fit the data well, though it over-predicted the overall average trade openness of countries by slightly more than the 50-country model. In addition, there are more zeros in bilateral trade data in the 60-country sample compared to the 50-country one. (With 50 countries, among the 2500 possible unidirectional bilateral trade flows, only 18 are zeros.) For these reasons we confine our analysis to the largest 50 countries.

<sup>18</sup>An interesting question is how large is the role of international trade in generating this welfare gain. To get a sense of this, we calculated the gains from the same reduction in fixed costs of entry under the assumption that each

The intuition for this result is that the distribution of firm size contains information on the relative importance of the marginal and the inframarginal varieties. Under Zipf's Law, the inframarginal varieties – the very large firms – are overwhelmingly more important than the marginal varieties. Thus, since the high entry costs do not affect the entry decision of the very large firms, they do not have much impact on welfare. As our quantitative exercise demonstrates, this is true even in a model that features a substantial intermediate input multiplier. As we move away from Zipf's Law, the distribution of firm size becomes flatter. As a result, entry of the marginal firms, and consequently the fixed costs of entry, become more important for welfare.

### 3.3 Counterfactual II: Reduction in Trade Barriers

Consider a global reduction in trade costs  $\tau_{ij}$ . How will it affect welfare, and what will be the relative importance of the intensive and the extensive margins? We know that welfare in this model is proportional to real income,  $W_i = w_i/P_i$ . From equation (13), welfare can be expressed, up to a constant that is the same in all countries and trade regimes, as follows:

$$W_i = \left[ \sum_{j=1}^N n_j \left( \tau_{ij} \frac{c_j}{c_i} \right)^{1-\varepsilon} G(a_{ij})^{\frac{\theta-(\varepsilon-1)}{\theta}} \right]^{\frac{1}{\beta(\varepsilon-1)}}. \quad (15)$$

A reduction in trade costs will impact the intensive margin, by making existing goods cheaper. That is captured by the  $\tau_{ij} \frac{c_j}{c_i}$  term. Additionally, welfare will increase due to the extensive margin, by leading to a greater number of varieties. This is captured by the  $G(a_{ij})$  term. Using a Taylor expansion, we can write the proportional increase in welfare as a function of the two margins:

$$\frac{\Delta W_i}{W_i} \approx \frac{1}{\beta} \sum_{j=1}^N \omega_{ij} \left[ \underbrace{-\frac{\Delta \left( \tau_{ij} \frac{c_j}{c_i} \right)}{\tau_{ij} \frac{c_j}{c_i}}}_{\text{Intensive Margin}} + \underbrace{\frac{\theta - (\varepsilon - 1)}{\theta} \frac{\Delta G(a_{ij})}{G(a_{ij})}}_{\text{Extensive Margin}} \right], \quad (16)$$

where  $\omega_{ij}$  is the weight of country  $j$  in country  $i$ 's price level:

$$\omega_{ij} \equiv \frac{n_j (\tau_{ij} c_j)^{1-\varepsilon} G(a_{ij})^{\frac{\theta-(\varepsilon-1)}{\theta}}}{\sum_{l=1}^N n_l (\tau_{il} c_l)^{1-\varepsilon} G(a_{il})^{\frac{\theta-(\varepsilon-1)}{\theta}}}.$$

It is immediate from (16) that the extensive margin does not have much of a chance to impact welfare. Any given change in the mass of new firms,  $\frac{\Delta G(a_{ij})}{G(a_{ij})}$ , while it may be large, is pre-multiplied by the term  $\frac{\theta-(\varepsilon-1)}{\theta(\varepsilon-1)}$ , which goes to zero as the economy becomes more granular. As we saw above, the calibrated value of this ratio is about 0.01.

country is in autarky. It turns out that the magnitude of the autarky gains is very similar. For instance, under the granular calibration the autarky gain is 3.47%, compared to 3.26% in the baseline open economy case. We conjecture that the average autarky percentage gain is slightly higher because in the absence of the possibility of importing, it is more important to have access to the most domestic varieties.

Table 3 reports the quantitative results for our sample of countries. A 10% reduction in trade barriers leads to an average increase in welfare of about 4.3%, with a range between 0.28 and 8.26%. Notably, this is somewhat higher than welfare gain we saw following a complete harmonization of entry barriers across countries. It turns out that the intensive margin accounts for 97.6% of the overall welfare gain. The table breaks down the extensive margin into the component coming from the new foreign varieties, and the component due to the disappearance of some domestic ones. The foreign extensive margin contributes 3.6% of the total welfare gain. It is partially undone by the domestic extensive margin, which is negative. As we can see, in Zipf’s world, the extensive margin plays a minimal role relative to the intensive one.

It is important to emphasize that this result is not due to a small increase in the number of foreign varieties. In this experiment, the 10% reduction in  $\tau_{ij}$  leads to an average 28% increase in the number of imported foreign varieties in this set of countries. The extensive margin, as measured by the number of varieties, is quantitatively important. However, its contribution to welfare is not.

The bottom panel reports these results with the non-granular calibration. Two features are most striking. First, the overall gains from a 10% reduction in  $\tau_{ij}$  are tiny compared to the granular calibration. The average gains are only 0.28% (less than one third of one percent), with a maximum of 1.7%. This is 15 times lower than the same reduction in trade costs in the granular calibration. Second, the overall importance of the intensive margin is almost the same as in the granular calibration, 95.5%. At first glance this is surprising. But it turns out that the welfare impact of the foreign extensive margin is indeed much bigger than in the granular calibration, as expected. The foreign extensive margin contributes 9% of the total welfare gain, 2.5 times greater than in the granular calibration. However, the domestic extensive margin is also more important for welfare, contributing  $-4.5\%$  of the total impact. That is, the disappearance of existing domestic varieties that accompanies the drop in trade costs also has a greater (negative) welfare impact compared to the granular case. The two partially cancel out, leaving the relative importance of the intensive margin roughly unchanged.

The main results are presented graphically in Figure 4. On the x-axis is the power law exponent in firm size,  $\theta/(\varepsilon - 1)$ , which varies from 1.06 (Zipf’s Law calibration) to 2. The lines display the welfare impact of the two counterfactual experiments we consider: a 10% reduction in  $\tau_{ij}$  (solid line) and the complete harmonization in  $f_{ij}$  to their U.S. level. The figure illustrates the importance of the firm size distribution for our conclusions about welfare. In particular, it is clear that changes in variable costs matter *more* for welfare as the economy becomes more granular, while changes in fixed costs matter *less*.

### 3.4 Impact of Varying Intermediate Goods Share in Final Production

How important is the intermediate goods multiplier in our counterfactual exercises?<sup>19</sup> In order to examine this question, we vary  $\beta$ , which is set to 0.5 in the baseline (equal shares of labor and intermediate goods for final production). Table 4 presents the total welfare changes in the two counterfactual exercises for  $\beta$  equal to 0.33 (smaller share of labor/larger share of intermediate goods) and 0.67 (larger share of labor/smaller share of intermediate goods) for the granular simulations. We also include the baseline case for comparison. The welfare impact of the same firm entry cost and trade openness changes decreases monotonically as  $\beta$  increases (the intermediate goods share falls). Increasing the intermediate input share from 1/2 to 2/3 ( $\beta = 0.33$ ) increases the welfare gain from a reduction in fixed costs by about 50%. The same absolute change in  $\beta$  in the other direction ( $\beta = 0.67$ ) reduces the welfare gains by less than a percentage point relative to the baseline case. Finally, the last column of the table presents the case of no intermediate goods multiplier ( $\beta = 1$ , and therefore  $c = w$ ). We see that the welfare gains from a reduction in entry costs are reduced roughly in half compared to the baseline case. To summarize, the variation in the share of intermediates in production has an important effect on welfare. Though we do not pursue this point further here, our comparative statics suggest that gains from trade may differ substantially across countries depending on their export specialization: countries that specialize in industries requiring lots of (foreign) intermediates will gain from trade substantially more than countries that simply produce output using the domestic factors of production.

## 4 Conclusion

The world economy and world trade flows are dominated by very large firms. This paper studies the implications of this stylized fact for two related aspects of the economy: entry costs and the extensive margin of exports. Our conclusions about the welfare impact of higher entry barriers and the extensive margin of trade are very sensitive to the assumptions on the size distribution of firms. In a model calibrated to match the observed firm-size distribution, the welfare costs of entry barriers are low. By contrast, gains from reductions in trade costs are much higher than in a model that does not exhibit Zipf's Law in firm size. Finally, the extensive margin accounts for less than 3% of the overall gains from trade.

What should we take away from this exercise? Quantitative evidence cannot be used to argue

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<sup>19</sup>The idea of the intermediate goods multiplier in the closed-economy setting is due to Jones (2008), who in addition assumes that intermediate inputs are complements in production to get an even larger effect, and explain potentially all the variation in per-capita incomes across countries. In the multi-country model of production with endogenous varieties, it would not be possible to incorporate complementarities of inputs, since producers of varieties are monopolistically competitive, and their profit maximization problem is not well defined when the elasticity of substitution is less than 1. Thus, we adopt the setup in which the elasticity of substitution in production and consumption is the same.

that entry costs and the extensive margin of trade are not important for welfare. We can establish, however, that the canonical model of production and trade with endogenous variety cannot generate a significant welfare impact of entry barriers and the extensive margin, while at the same time matching both the empirically observed distribution of firm size and trade volumes. If these matter, it must be through some other channel. Uncovering the conditions under which the costs of entry into domestic and foreign markets matter more remains a fruitful avenue for future research.

In this context, our results have implications for the trade versus institutions debate. There is no consensus regarding the relative importance of trade openness and economic institutions in explaining income differences. Rodrik, Subramanian and Trebbi (2004) and Rigobon and Rodrik (2005) argue that once institutions are controlled for, trade openness has no impact on income. By contrast, Dollar and Kraay (2003) and Alcalá and Ciccone (2004) make the case that trade openness matters independently of institutions. The quantitative results in the paper imply that trade has a far greater impact than one particular form of institutions – entry barriers. A reduction in the variable trade costs of only 10% has the a greater welfare impact than even the complete elimination of entry barrier differences across countries. More generally, the results imply that to evaluate the relative importance of trade versus domestic regulation for welfare, we must carefully take into account the underlying structure of the economy.

## Appendix A Proof of Proposition 1

**Proof:** The total variable profits from selling to country  $j$  from country  $i$  are:

$$\Pi_{ji}^V = \int_{J_{ji}} \frac{X_j}{\varepsilon P_j^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ji} c_i a(k) \right)^{1-\varepsilon} dk.$$

The total sales from  $i$  to  $j$  are:

$$X_{ji} = \int_{J_{ji}} \frac{X_j}{P_j^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ji} c_i a(k) \right)^{1-\varepsilon} dk.$$

Therefore,  $\Pi_{ji}^V = \frac{X_{ji}}{\varepsilon}$ .

The total fixed costs paid by firms in country  $i$  to enter market  $j$  are equal to  $f_{ji} c_i n_i (b a_{ji})^\theta$ .

We need to show that this quantity is also a constant multiple of  $X_{ji}$ . To do so, write

$$\begin{aligned} X_{ji} &= \frac{X_j}{P_j^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ji} c_i \right)^{1-\varepsilon} \int_{J_{ji}} (a(k))^{1-\varepsilon} dk \\ &= \frac{X_j}{P_j^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ji} c_i \right)^{1-\varepsilon} n_j \frac{b^\theta \theta}{\theta - (\varepsilon - 1)} a_{ji}^{\theta - (\varepsilon - 1)} \\ &= \frac{X_j}{P_j^{1-\varepsilon}} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{ji} c_i \right)^{1-\varepsilon} n_j \frac{b^\theta \theta}{\theta - (\varepsilon - 1)} a_{ji}^\theta c_i f_{ji} \frac{\varepsilon P_j^{1-\varepsilon}}{X_j} \left( \frac{\varepsilon - 1}{\varepsilon} \right) \\ &= \frac{\theta}{\theta - (\varepsilon - 1)} \varepsilon n_j (b a_{ji})^\theta c_i f_{ji}. \end{aligned}$$

Therefore, the total fixed costs paid by firms in  $i$  to export to  $j$  are a constant multiple of  $X_{ji}$ :

$$n_j (b a_{ji})^\theta c_i f_{ji} = \frac{\theta - (\varepsilon - 1)}{\theta} \frac{X_{ji}}{\varepsilon}.$$

Therefore, the total profits from selling to  $j$  from country  $i$  are:

$$\begin{aligned} \Pi_{ji} &= \Pi_{ji}^V - \frac{\theta - (\varepsilon - 1)}{\theta} \frac{X_{ji}}{\varepsilon} \\ &= \frac{X_{ji}}{\varepsilon} \left( 1 - \frac{\theta - (\varepsilon - 1)}{\theta} \right) \\ &= X_{ji} \frac{(\varepsilon - 1)}{\varepsilon \theta}. \end{aligned}$$

This means that the total profits from selling to all countries equal:

$$\Pi_i = \sum_{j=1}^N \Pi_{ji} = \frac{(\varepsilon - 1)}{\varepsilon \theta} \sum_{j=1}^N X_{ji}.$$

Since in equilibrium total income equals total expenditure in each country,  $X_i = \sum_{j=1}^N X_{ji}$ , leading to the result that  $\Pi_i = \frac{(\varepsilon - 1)}{\varepsilon \theta} X_i$ . ■



## Appendix B Model with Market Penetration Costs

A recent contribution by Arkolakis (2008) emphasizes that the model with simple fixed costs of accessing markets is too stark. Instead, Arkolakis (2008) proposes a model in which firms choose not only whether to enter a particular market, but what share of the consumers in that market to serve. Arkolakis (2008) and Eaton et al. (2008) demonstrate that modeling entry costs in this more continuous way is important to account for the empirical regularity that many firms export only small amounts abroad. In this Appendix, we extend the baseline model to feature market penetration costs instead of fixed entry costs, and demonstrate that the total welfare in such a model differs from the baseline only by a constant. As a result, in any policy experiment the market penetration costs model produces welfare changes that are identical to the baseline fixed costs model.

Our functional form assumption follows Eaton et al. (2008). Assume that rather than paying the fixed cost  $f_{ij}c_j$  to gain access to all consumers in market  $i$ , a firm in country  $j$  incurs a cost

$$f_{ij}c_j \frac{1 - (1-s)^{1-\frac{1}{\lambda}}}{1 - \frac{1}{\lambda}}$$

to reach a share  $s$  of consumers in that market. Given the demand for its variety by the consumer reached in country  $i$ , the firm with marginal cost  $a(k)$  from country  $j$  maximizes its profits by choosing both its price and market penetration  $s_i(k)$  optimally. The profits are given by:

$$\pi_i(k) = [p_i(k) - \tau_{ij}c_j a(k)] \left( \frac{p_i(k)}{P_i} \right)^{-\varepsilon} s_i(k) X_i - f_{ij}c_j \frac{1 - (1-s)^{1-\frac{1}{\lambda}}}{1 - \frac{1}{\lambda}},$$

where the price index,  $P_i$ , now aggregates over the prices of varieties available to a typical consumer in  $i$ , and not over all the varieties that are sold in that country. It is easily verified that the price is still a constant markup over the marginal cost. Optimal market penetration for a firm with marginal cost  $a(k)$  is given by:

$$s_i(k) = 1 - \left[ \frac{X_i}{\varepsilon c_j f_{ij}} \left( \frac{\frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k)}{P_i} \right)^{1-\varepsilon} \right]^{-\lambda}. \quad (\text{B.1})$$

Finally, the firm will only enter market  $i$  if at zero market penetration, profits are increasing in  $s$ :  $\frac{\partial \pi_i(k)}{\partial s} |_{s=0} > 0$ . It turns out that the cutoff  $a_{ij}$  for positive sales from  $j$  to  $i$  has the exact same form as in the baseline model, and is given by equation (4). That expression can be combined with equation (B.1) to write the sales of a firm with marginal cost  $a(k)$  from country  $j$  to country  $i$  as:

$$\left[ 1 - \left( \frac{a(k)}{a_{ij}} \right)^{\lambda(\varepsilon-1)} \right] \left( \frac{\frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k)}{P_i} \right)^{1-\varepsilon} X_i.$$

As first observed by Arkolakis (2008), the baseline model with simple fixed costs provides the best approximation to the sales of the largest firms: as the marginal cost  $a(k)$  decreases,

$s_i(k) = \left[ 1 - \left( \frac{a(k)}{a_{ij}} \right)^{\lambda(\varepsilon-1)} \right]$  approaches 1 and the firm penetrates the entire market. This result does not rely on the Zipf's Law assumption: the market penetration ratio  $s_i(k)$  does not depend on the combination of parameters  $\frac{\theta}{\varepsilon-1}$ . As we argue at the end of this section, Zipf's Law does imply that the large firms are the ones most important for welfare, and thus the assumption of simple fixed costs adopted in the main text will not substantially affect our conclusions.

Under the Pareto distribution of productivity draws, the expression for the price level in country  $i$  is given by:

$$\begin{aligned}
P_i^{mp} &= \left( \sum_{j=1}^N n_j \int_0^{a_{ij}} \left[ \frac{\varepsilon}{\varepsilon-1} \tau_{ij} c_j a(k) \right]^{1-\varepsilon} s_j(k) dG(a(k)) \right)^{\frac{1}{1-\varepsilon}} \\
&= \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} - \frac{\theta}{\theta - (\varepsilon - 1)(1 - \lambda)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{X_i}{\varepsilon} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \times \\
&\quad \left( \sum_{j=1}^N n_j \left( \frac{1}{\tau_{ij} c_j} \right)^\theta \left( \frac{1}{c_j f_{ij}} \right)^{\frac{\theta - (\varepsilon - 1)}{\varepsilon - 1}} \right)^{-\frac{1}{\theta}}. \tag{B.2}
\end{aligned}$$

Comparing equations (6) and (B.2), it is clear that the price levels in the baseline model and the market penetration cost model differ only by a constant. The rest of the solution is unchanged. In particular, it is straightforward to show that Proposition 1 still holds, and that the wages are still determined by equation (9). Thus, the solution to the market penetration costs model proceeds to find  $w_i^{mp}$  and  $P_i^{mp}$  for all  $i = 1, \dots, N$  that solve the system of equations given by (9) and (B.2). We now state the main result of this Appendix.

**Proposition 2** *Let the vectors  $[w_1, \dots, w_N]$  and  $[P_1, \dots, P_N]$  jointly be a solution to the system of equations defining the equilibrium in the baseline fixed costs model, (6) and (9). Then, the vectors*

$$[w_1^{mp}, \dots, w_N^{mp}] = [w_1, \dots, w_N] \tag{B.3}$$

and

$$[P_1^{mp}, \dots, P_N^{mp}] = \delta [P_1, \dots, P_N] \tag{B.4}$$

are a solution to the system of equations (B.2) and (9) that define the equilibrium in the market penetration costs model.

**Proof:** It is immediate from examining (9) that the vector  $[w_1, \dots, w_N]$  that solves (9) is the same under  $[P_1, \dots, P_N]$  and  $[P_1^{mp}, \dots, P_N^{mp}]$  when the latter is defined by (B.4), since  $\delta$  cancels out from the numerator and the denominator. We now show that as long as (B.3) is satisfied, (B.4) holds as well for some constant  $\delta$ . The vector  $[P_1^{mp}, \dots, P_N^{mp}]$  provides a solution to the market penetration

costs model if  $\forall i$ , (B.2) holds. We check directly whether the vector  $\delta [P_1, \dots, P_N]$  satisfies that condition:

$$P_i^{mp} = \delta P_i = \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} - \frac{\theta}{\theta - (\varepsilon - 1)(1 - \lambda)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{w_i L_i}{\varepsilon \beta \left(1 - \frac{\varepsilon - 1}{\theta \varepsilon}\right)} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \times \left( \sum_{j=1}^N n_j \left( \frac{1}{\tau_{ij} w_j^\beta (\delta P_j)^{1 - \beta}} \right)^\theta \left( \frac{1}{w_j^\beta (\delta P_j)^{1 - \beta} f_{ij}} \right)^{\frac{\theta - (\varepsilon - 1)}{\varepsilon - 1}} \right)^{-\frac{1}{\theta}}. \quad (\text{B.5})$$

After rearranging it becomes:

$$\delta^{\beta - (1 - \beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} P_i = \frac{1}{b} \left[ \frac{\theta}{\theta - (\varepsilon - 1)} - \frac{\theta}{\theta - (\varepsilon - 1)(1 - \lambda)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{w_i L_i}{\varepsilon \beta \left(1 - \frac{\varepsilon - 1}{\theta \varepsilon}\right)} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \times \left( \sum_{j=1}^N n_j \left( \frac{1}{\tau_{ij} w_j^\beta P_j^{1 - \beta}} \right)^\theta \left( \frac{1}{w_j^\beta P_j^{1 - \beta} f_{ij}} \right)^{\frac{\theta - (\varepsilon - 1)}{\varepsilon - 1}} \right)^{-\frac{1}{\theta}},$$

which is the same as (6) for  $\delta$  satisfying  $\left[ \frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-\frac{1}{\theta}} = \left[ \frac{\theta}{\theta - (\varepsilon - 1)} - \frac{\theta}{\theta - (\varepsilon - 1)(1 - \lambda)} \right]^{-\frac{1}{\theta}} \delta^{-\left(\beta - (1 - \beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}\right)}$ . Since the vector  $[P_1, \dots, P_N]$  satisfies (6), we have shown that  $\delta [P_1, \dots, P_N]$  satisfies (B.5), which completes the proof. ■

The main consequence of Proposition 2 is that the total welfare in the market penetration costs model differs from the welfare in the basic fixed costs model only by a constant:  $w_i^{mp}/P_i^{mp} = (1/\delta)w_i/P_i$ . This implies that any percentage change in welfare calculated in this model will be identical to the baseline in the main text.

One additional remark is worth making on the relationship between the market penetration costs model and this paper. Straightforward rearranging yields the following expression for  $\delta$ :

$$\delta = \left[ \frac{\lambda(\varepsilon - 1)}{\theta - (\varepsilon - 1)(1 - \lambda)} \right]^{-\frac{1}{\theta} \frac{1}{\beta - (1 - \beta) \frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}}}$$

Setting  $\lambda = 1$  the expression in the square brackets becomes  $(\varepsilon - 1)/\theta$ .<sup>20</sup> Therefore, it is immediate that as we approach Zipf's Law,  $\delta \rightarrow 1$  and the welfare *level* in the market penetration cost model converges exactly to the welfare level in the simple fixed costs model. This is intuitive: under Zipf's Law, what matters the most for welfare are the biggest firms, for which the market penetration margin matters the least, since they choose to serve the entire market.

<sup>20</sup>This is the value of  $\lambda$  preferred by Arkolakis (2008). Using Simulated Method of Moments, Eaton et al. (2008) indeed estimate a value of  $\lambda = 0.91$  with a standard error of 0.12. This type of value for  $\lambda$  implies a fair amount of curvature to the market penetration costs, and thus many firms that choose to penetrate only a small share of the export market. The fixed cost model obtains instead when  $\lambda = \infty$ .

## Appendix C Power Laws in Firm Size in the ORBIS/AMADEUS Database

This Appendix uses a large cross-country firm-level database to assess whether Zipf’s Law approximates well the distribution of firm size in a large sample of countries. Though we use the largest available non-proprietary firm-level database in this analysis, the results should be interpreted with caution: coverage is quite uneven across countries and years, implying that power law estimates may not be reliable or comparable across countries. Nonetheless, as we describe below, Zipf’s Law provides a good approximation for the firm size distribution in most countries in this sample.

We estimate power laws in firm size using ORBIS, a large multi-country database published by Bureau van Dijk that contains information on more than 50 million companies worldwide. The data come from a variety of sources, including, but not limited to, registered filings and annual reports. Coverage varies by world region: there are data on some 17 million companies in the U.S. and Canada, 22 million companies in the 46 European countries, 6.2 million companies from Central and South America, 5.3 million from Asia, but only 260,000 from Africa and 45,000 from the Middle East. Importantly, the database includes both publicly traded and privately held firms. For 41 European countries, the AMADEUS database also published by Bureau van Dijk contains similar information but often has better coverage (more firms). In addition, the data in AMADEUS appear more standardized across countries. Thus, for countries with better coverage and data quality in AMADEUS compared to ORBIS, we use information from the former database. While in principle data are available going back to mid-1990s for some countries, coverage improves dramatically for more recent years. For this reason, we focus our analysis on 2006, the year with the most observations available. The main variable used in the analysis is total sales. We restrict our empirical analysis to countries that have sales figures for at least 1000 firms in 2006.

In order to obtain reliable estimates, this paper uses three standard methods of estimating the slope of the power law  $\zeta$ . The first method, based on Axtell (2001), makes direct use of the definition of the power law (10), which in natural logs becomes:

$$\log(\Pr(x > s)) = \log(c) - \zeta \log(s). \tag{C.1}$$

For a grid of values of sales  $s$ , the estimated probability  $\Pr(x > s)$  is simply the number of firms in the sample with sales greater than  $s$  divided by the total number of firms. We then regress the natural log of this probability on  $\log(s)$  to obtain our first estimate of  $\zeta$ . Following the typical approach in the literature, we do this for the values of  $s$  that are equidistant from each other on log scale. This implies that in absolute terms, the intervals containing low values of  $s$  are narrower than the intervals at high values of  $s$ . This is done to get a greater precision of the estimates: since there are fewer large firms, observations in small intervals for very high values of  $s$  would be more

noisy.

The second approach starts with the observation that the cdf in (10) has a probability density function

$$f(s) = c\zeta s^{-(\zeta+1)}. \quad (\text{C.2})$$

To estimate this pdf, we divide the values of firm sales into bins of equal size on the log scale, and compute the frequency as the number of firms in each bin divided by the width of the bin. Since in absolute terms the bins are of unequal size, we regress the resulting frequency observations on the value of  $s$  which is the geometric mean of the endpoints of the bin (this approach follows Axtell 2001). Note that the resulting coefficient is an estimate of  $-(\zeta + 1)$ .<sup>21</sup>

Table A1 reports the results. The left panel reports estimates of equation (C.1), the right panel, equation (C.2). (Note that the right panel's estimates are of  $-(\zeta + 1)$ , thus they should differ from the right panel by about  $-1$ .) The columns report the power law coefficient, the  $R^2$ , and the  $p$ -value of the test that the coefficient differs from  $-1$  ( $-2$  in the right panel). Several things are worth noting about these results. First, the power law approximates the data well: with the exception of the U.K., the  $R^2$ 's are all well above 0.9. Second, most of the power law coefficients are very close to 1 in absolute value, and many are not statistically different from  $-1$ . Those that are statistically different from  $-1$  tend to be *lower* in absolute value, implying that if the firm size distribution follows a power law in those countries, it is even more fat-tailed than Zipf. Aside from the U.K. – whose firm-size distribution does not appear to be well approximated by a power law, at least in this database – the least fat-tailed countries have the power law exponent of about  $-1.2$ , still quite far from  $-2$  and thus comfortably within the granular range. Finally, the country sample is diverse: it includes major European economies (France, Germany, Netherlands), smaller E.U. accession countries (Czech Republic, Estonia), major middle income countries (Brazil, Argentina), as well as the two largest emerging markets (India and China).

It is important to note that these results do not establish that the distribution of firm size in these countries follows a power law, as opposed to some other distribution. Indeed, as noted by Gabaix (2008), with more parameters (allowing for more curvature), one will always fit the data better. Rather, Gabaix (2008) suggests that what is important is whether a power law provides a good fit to the data, which appears to be the case in our results.

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<sup>21</sup>Finally, we also regressed  $\log(\text{rank} - 1/2)$  of each firm in the sales distribution on  $\log$  of its sales. This is the estimator suggested by Gabaix and Ibragimov (2009), which delivers very similar results. If anything, the power law exponents implied by this estimator are even lower in absolute value than those reported in this Appendix.

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**Table 1.** Top 49 Countries and the Rest of the World in Terms of 2004 GDP

Country	GDP/ World GDP	Country	GDP/ World GDP
United States	0.300	Indonesia	0.006
Japan	0.124	South Africa	0.006
Germany	0.076	Norway	0.006
France	0.054	Poland	0.005
United Kingdom	0.044	Finland	0.005
Italy	0.041	Greece	0.004
China	0.028	Venezuela, RB	0.004
Canada	0.026	Thailand	0.004
Brazil	0.021	Portugal	0.003
Spain	0.020	Colombia	0.003
India	0.017	Nigeria	0.003
Australia	0.016	Algeria	0.003
Russian Federation	0.015	Israel	0.003
Mexico	0.015	Philippines	0.003
Netherlands	0.015	Malaysia	0.002
Korea, Rep.	0.011	Ireland	0.002
Sweden	0.010	Egypt, Arab Rep.	0.002
Switzerland	0.010	Pakistan	0.002
Belgium	0.009	Chile	0.002
Argentina	0.008	New Zealand	0.002
Saudi Arabia	0.007	Czech Republic	0.002
Austria	0.007	United Arab Emirates	0.002
Iran, Islamic Rep.	0.007	Hungary	0.002
Turkey	0.007	Romania	0.002
Denmark	0.006	Rest of the World	0.027

Notes: Ranking of top 49 countries and the rest of the world in terms of 2004 U.S.\$ GDP. We include Hong Kong, POC, and Singapore in Rest of the World. Source: The World Bank (2007b).

**Table 2.** Bilateral Trade Shares: Data and Model Predictions for the 50-Country Sample

	model	data
Domestic sales as a share of domestic absorption ( $\pi_{ii}$ )		
mean	0.7228	0.7555
median	0.7256	0.7982
corr(model, data)	0.4757	
Export sales as a share of domestic absorption ( $\pi_{ij}$ )		
mean	0.0056	0.0047
median	0.0025	0.0011
corr(model, data)	0.7107	

Notes: This table reports the means and medians of domestic output (top panel), and bilateral trade (bottom panel), both as a share of domestic absorption, in the model and in the data. Source: International Monetary Fund (2007).

**Table 3.** Welfare Gains

$\frac{\theta}{\varepsilon-1} = 1.06$ (Zipf's World)					
Counterfactual	Total change in welfare	Intensive margin	Extensive margin		
Complete harmonization of entry costs	3.26 (0.03, 7.32)			Foreign	Domestic
				0.15	-0.05
10% reduction in $\tau$	4.33 (0.28, 8.26)	4.23 (0.28, 8.06)		(0.05, 0.27)	(-0.07, -0.01)
		<b>0.976</b>		<b>0.036</b>	<b>-0.013</b>
$\frac{\theta}{\varepsilon-1} = 2$					
Counterfactual	Total change in welfare	Intensive margin	Extensive margin		
Complete harmonization of entry costs	40.87 (0.00, 104.78)			Foreign	Domestic
				0.025	-0.013
10% reduction in $\tau$	0.28 (0.01, 1.72)	0.27 (0.01, 1.64)		(0.000, 0.137)	(-0.057, 0.000)
		<b>0.955</b>		<b>0.090</b>	<b>-0.045</b>

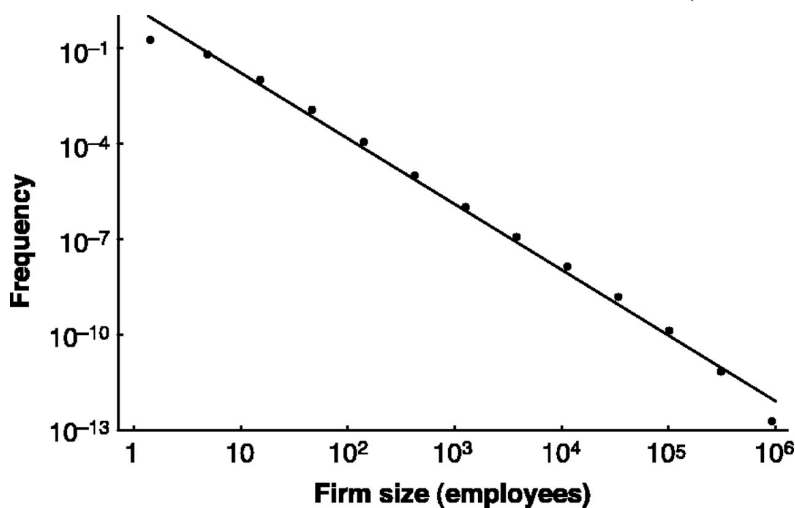
Notes: This table reports the welfare increase, in percentage points, due to each counterfactual experiment. The numbers in parentheses indicate the range across the 50 countries in the sample. The numbers in bold give the share of each margin (intensive, foreign extensive, and domestic extensive) in the total welfare impact.

**Table 4.** Welfare Gains when Varying Share of Intermediate Goods in Final Production

Counterfactual	Total change in welfare			
	$\beta = 0.33$	$\beta = 0.5$ (baseline)	$\beta = 0.67$	$\beta = 1$
Complete harmonization of entry costs	5.03 (0.06, 11.37)	3.26 (0.03, 7.32)	2.41 (0.02, 5.39)	1.60 (0.00, 3.58)
10% reduction in $\tau$	4.79 (0.22, 8.87)	4.33 (0.28, 8.26)	4.07 (0.32, 7.85)	3.80 (0.37, 7.37)

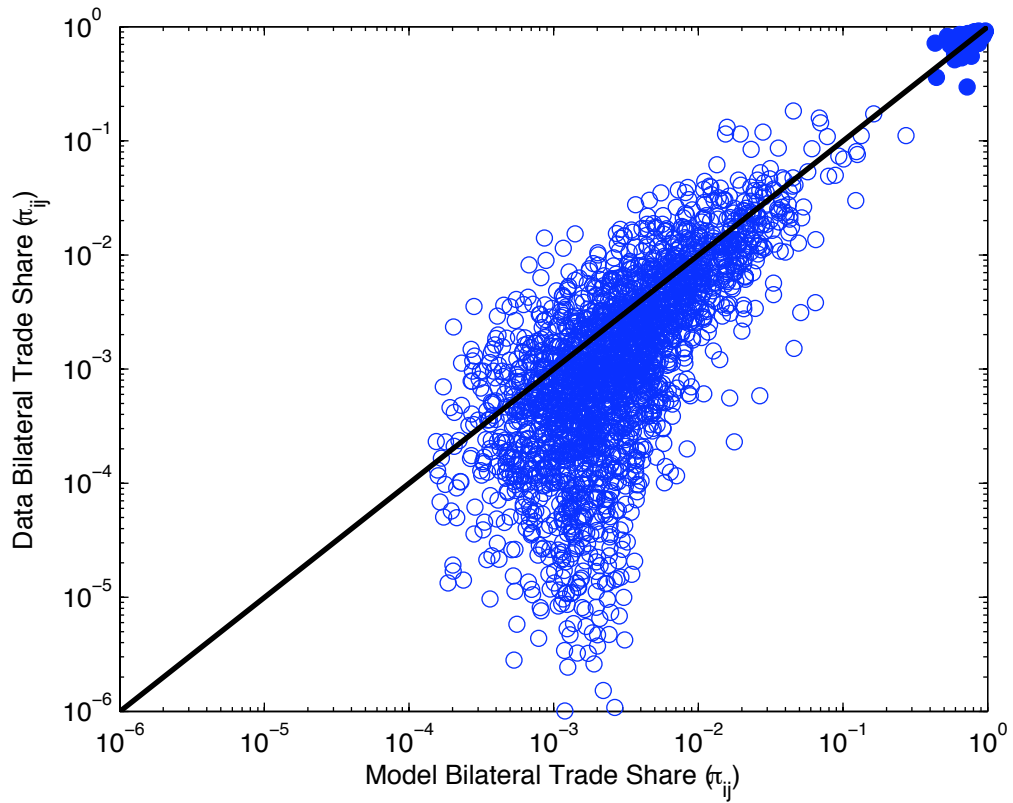
Notes: This table reports the welfare increase, in percentage points, due to each counterfactual experiment. The numbers in parentheses indicate the range across the 50 countries in the sample.  $1 - \beta$  equals the share of intermediate goods in final production. These counterfactuals are done assuming  $\frac{\theta}{\varepsilon-1} = 1.06$  (Zipf's World).

**Figure 1.** Estimated Power Law in Firm Size in the U.S. (Axtell, 2001).



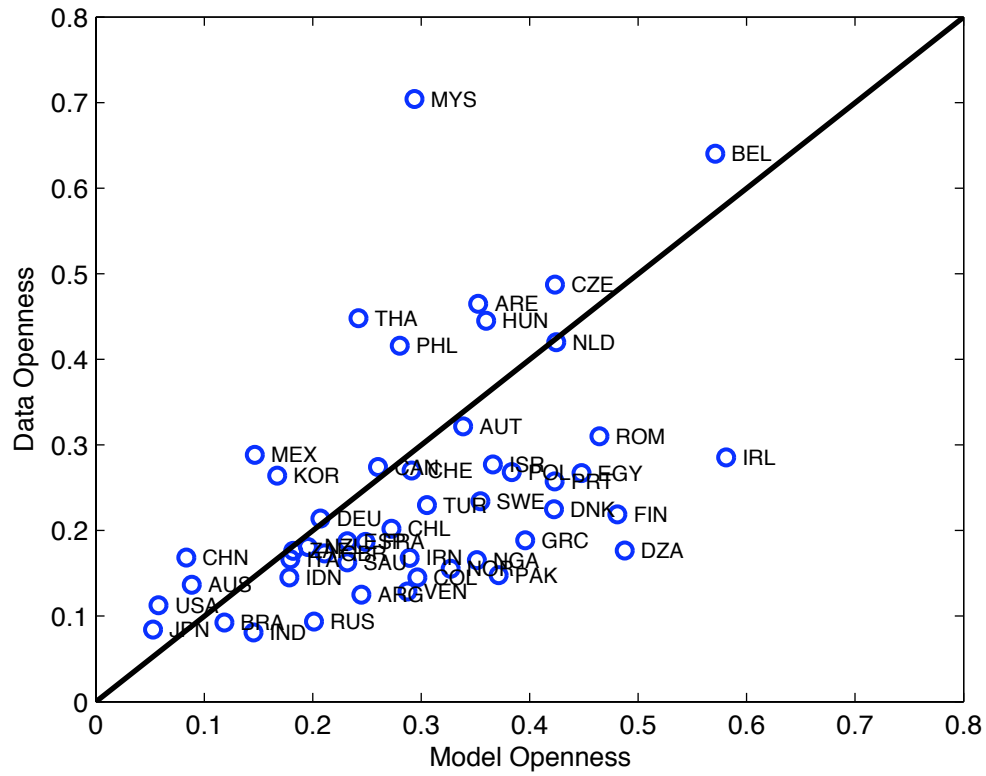
Notes: Reproduced from Axtell (2001). This figure depicts the power law in firm size in the U.S.: it plots the log frequency of the firms against log of firm size, measured by the number of employees. The solid line is the OLS regression fit through the data. The estimated slope coefficient is -2.059 (s.e. 0.054), which implies  $\zeta = 1.059$ . The adjusted  $R^2$  is 0.992. Similar relationships are also reported for sales.

**Figure 2.** Bilateral Trade Shares: Data and Model Predictions



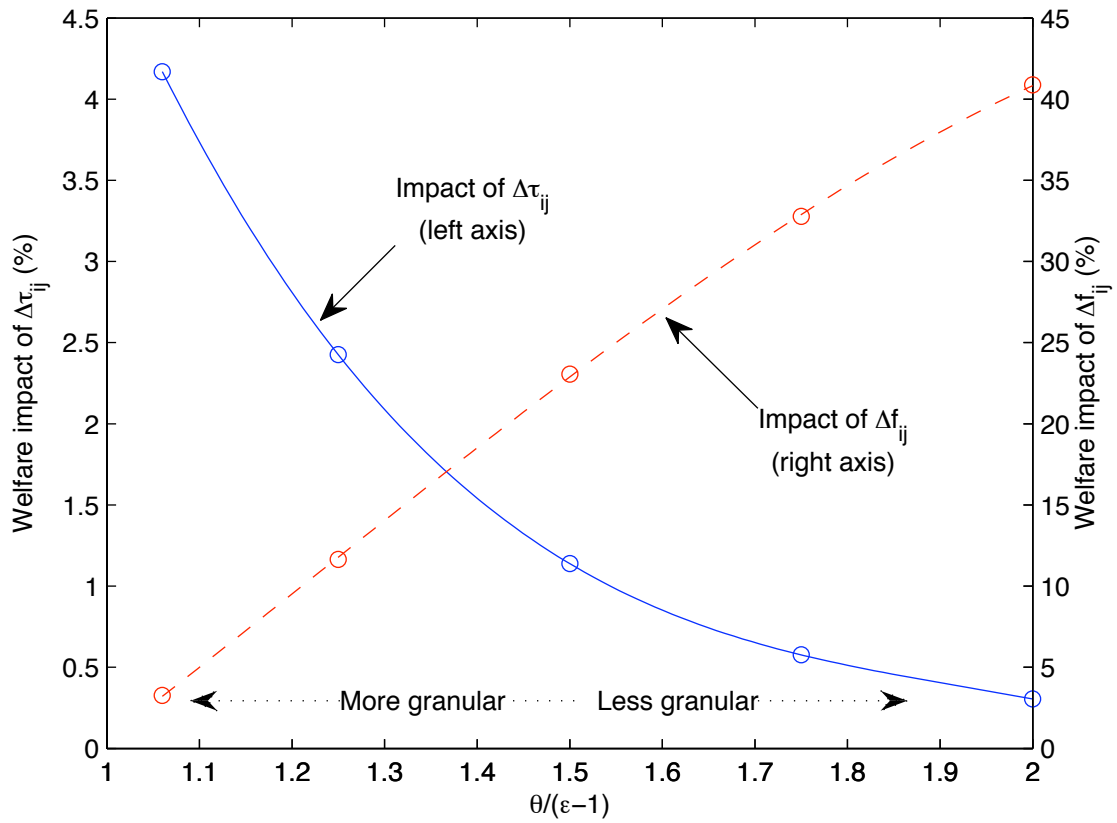
Notes: This figure reports the scatterplot of domestic output ( $\pi_{ii}$ ) and bilateral trade ( $\pi_{ij}$ ), both as a share of domestic absorption. The values implied by the model are on the horizontal axis. Actual values are on the vertical axis. Solid dots represent observations of  $\pi_{ii}$ , while hollow dots represent bilateral trade observations ( $\pi_{ij}$ ). The line through the data is the 45-degree line.

**Figure 3.** Trade Openness: Data and Model Predictions



Notes: This figure reports total imports as a share of domestic absorption ( $1 - \pi_{ii}$ ). The values implied by the model are on the horizontal axis. Actual values are on the vertical axis. The line through the data is the 45-degree line.

**Figure 4.** The Welfare Impact of Reductions in Fixed and Variable Costs and the Size Distribution of Firms



Notes: This figure reports the percentage changes in welfare due to a reduction in iceberg trade costs (solid line, left axis) and a reduction in fixed costs of entry (dashed line, right axis), as a function of the distribution of firm size.

**Table A1.** Country-by-Country Estimates of Power Laws in Firm Size

Country	CDF Estimation			PDF Estimation		
	PL Coef.	$R^2$	$p$ -value	PL Coef.	$R^2$	$p$ -value
Austria	-1.207**	0.993	0.0000	-2.160**	0.998	0.0002
Belgium	-0.724**	0.952	0.0002	-1.662**	0.987	0.0001
Germany	-1.035**	0.985	0.3767	-1.998**	0.996	0.9528
Denmark	-0.627**	0.938	0.0000	-1.626**	0.985	0.0001
Estonia	-0.739**	0.962	0.0001	-1.591**	0.992	0.0000
Finland	-0.815**	0.988	0.0000	-1.747**	0.999	0.0000
France	-0.960**	0.993	0.1420	-1.959**	0.998	0.1066
U.K.	-1.349**	0.704	0.2133	-2.265**	0.870	0.3604
Greece	-1.018**	0.987	0.6112	-1.965**	0.997	0.2878
Croatia	-0.719**	0.924	0.0009	-1.675**	0.978	0.0014
Ireland	-0.662**	0.966	0.0000	-1.702**	0.923	0.0686
Italy	-1.009**	0.990	0.7647	-2.041**	0.996	0.3070
Netherlands	-0.891**	0.979	0.0175	-1.845**	0.990	0.0193
Norway	-0.852**	0.995	0.0000	-1.838**	0.997	0.0002
Slovakia	-1.161**	0.981	0.0068	-2.071**	0.984	0.3919
Sweden	-0.861**	0.998	0.0000	-1.874**	0.999	0.0001
Argentina	-0.977**	0.998	0.1384	-1.938**	0.999	0.0185
Brazil	-1.107**	0.977	0.0599	-2.051**	0.992	0.3785
Canada	-0.969**	0.999	0.0200	-1.968**	0.999	0.1847
Switzerland	-0.826**	0.995	0.0000	-1.782**	0.999	0.0000
China	-1.117**	0.985	0.0159	-2.084**	0.998	0.0253
Czech Republic	-0.915**	0.973	0.0934	-1.929**	0.989	0.2835
Spain	-0.955**	0.996	0.0318	-1.963**	0.998	0.1931
Hungary	-0.975**	0.995	0.2407	-1.933**	0.998	0.0347
Lithuania	-1.189**	0.982	0.0024	-2.129**	0.990	0.0723
Latvia	-1.201**	0.985	0.0008	-2.133**	0.993	0.0359
Portugal	-1.035**	0.989	0.2987	-1.993**	0.998	0.8170
Russia	-1.033**	0.996	0.1317	-2.016**	0.997	0.6782
Ukraine	-0.940**	0.994	0.0190	-1.909**	0.999	0.0011
Australia	-1.105**	0.991	0.0064	-2.018**	0.982	0.8300
India	-0.870**	0.978	0.0066	-1.816**	0.993	0.0020
Japan	-0.957**	0.993	0.1052	-1.989**	0.995	0.8035
Korea	-0.893**	0.997	0.0000	-1.873**	0.998	0.0004
Mexico	-0.616**	0.982	0.0000	-1.623**	0.993	0.0000
Malaysia	-0.930**	0.980	0.1030	-1.789**	0.993	0.0006
Singapore	-0.828**	0.973	0.0017	-1.774**	0.993	0.0004
Thailand	-0.920**	0.958	0.1993	-1.807**	0.996	0.0004
Taiwan POC	-0.830**	0.975	0.0014	-1.763**	0.994	0.0001

Notes: \*\* – significant at the 1% level. This table reports the estimated of power laws in firm size across countries. Column “PL Coef.” reports the coefficient on the power law for each country, the second column reports the  $R^2$ , the third column reports the  $p$ -value of the test that the power law coefficient is statistically different from  $-1$  ( $-2$  in the right panel). The estimates are based on 2006 firm-level sales data from ORBIS/AMADEUS. Variable definitions, sources, and estimation techniques are described in detail in the text.