RESEARCH SEMINAR IN INTERNATIONAL ECONOMICS

School of Public Policy
The University of Michigan
Ann Arbor, Michigan 48109-1220

Discussion Paper No. 443

Patterns of Trade and Growth across Cones

Alan V. Deardorff
University of Michigan

July 23, 1999

Recent RSIE Discussion Papers are available on the World Wide Web at:
http://www.spp.umich.edu/rsie/workingpapers/wp.html

Patterns of Trade and Growth across Cones

Alan V. Deardorff

The University of Michigan

Paper prepared for the conference,
"Dynamics, Economic Growth, and International Trade IV"
Tilburg, the Netherlands
July 9-10, 1999

Revised
July 23, 1999

ABSTRACT

Patterns of Trade and Growth across Cones

Alan V. Deardorff

The University of Michigan

This paper examines the implications of the Heckscher-Ohlin (HO) Model for the patterns of production and trade that will emerge as a country grows. It focuses primarily on world equilibria that include two or more cones of diversification. Starting with the textbook model of two factors and two goods, growth paths for production and trade are derived in terms of a country's capital-labor ratio relative to that of the world. With additional goods and countries, multiple cones create a ladder of comparative advantage that a country will climb as it accumulates capital relative to the world. With additional factors as well, more complicated patterns can emerge. In a three-factor model based on Krueger (1977), a country with fixed land, growing labor, and faster growing capital can first work its way down the ladder of comparative advantage before climbing back up. Using a graphical representation of a more general three-factor model due to Leamer (1987), cones of diversification with large numbers of goods take the form of polygons that a growing country may pass through, then cross between. In all cases, the lesson of the HO Model is that growth causes repeated and extreme changes in patterns of specialization and trade over time.

Keywords: Trade and Growth
Multi-Cone Models

JEL Subject Code:
F11 Neoclassical Trade Models

Correspondence:

Alan V. Deardorff
Department of Economics
University of Michigan
Ann Arbor, MI 48109-1220

Tel. 734-764-6817
Fax. 734-763-9181
E-mail: alandear@umich.ed
http://www.econ.lsa.umich.edu/~alandear/

July 23, 1999

Patterns of Trade and Growth across Cones [*]

Alan V. Deardorff
The University of Michigan

I. Introduction

The purpose of this paper is to examine the implications of the Heckscher-Ohlin (HO) trade model for the patterns of trade across time as countries grow. This is hardly a new issue, and others, whose work I will simply review, have already provided some of the most important insights about it. However, recent developments in trade theory have expanded our understanding of the HO model and of the kinds of world equilibrium that are most likely to arise in the world of very diverse factor endowments – the equilibria with multiple cones of diversification. It therefore seems worthwhile to review more carefully what the HO model implies in such a world about the effects of growth on the patterns of specialization and trade.

For this purpose, I will not attempt to explain growth at all. Others – including myself in other papers – have explored the interactions between trade and growth in both neoclassical and endogenous growth models. The primary purpose has usually been to determine how trade may matter for growth. But mine is the older question of how growth matters for trade. For that I will simply assume that countries do grow, in the sense of expanding their endowments of capital (human or physical) relative to labor.

[*] I would like to thank the participants in the Tilburg conference, where this paper was presented, for their helpful comments.

Growth may also arise, more usefully perhaps, from technological progress, but for the most part I will ignore that source of growth. If technology expands for the world as a whole, as seems most likely, then it simply superimposes a trend on the changing patterns that arise from factor accumulation. If, on the other hand, technology expands differently in different countries, then just about anything can happen to trade patterns depending on the nature of that expansion. However, to the extent that technological progress is equivalent simply to improvements in the efficiency of factors, then it is also equivalent to the accumulation of factors that is the subject of my analysis.

I will begin in section II by recalling the implications of the simplest HO model: With just two factors, two goods, and two countries, how does the HO model predict trade patterns to evolve over time as one or both countries grow? Section III will then expand the numbers of both goods and countries, examining first the kinds of static equilibrium that are possible in that context, and second how a single country's trade pattern will behave over time within such a world equilibrium if it remains fixed. Of course, with all or most countries growing, the world equilibrium will not remain fixed, so I will also look at how the world equilibrium will change over time, and then reformulate the conclusions about single countries in terms of their growth relative to a changing world.

All of this is with only two factors, which is often the most that can be handled conveniently with the tools at hand. In section IV, however, I will recall two contributions from earlier literature that allow us to say something in particular models with more than two factors. The first is an old model by Krueger (1977), one that I previously put through its paces in Deardorff (1984), where a third factor, land, is used in only one sector of a many-good economy. The second is an ingenious depiction of a more general three-factor model by Leamer (1987). I will report what both of these models have to say about effects of growth on trade patterns, then try to say a bit more on my own.

II. The Textbook Model

Consider the simple 2×2×2 textbook HO model. Two factors, capital K and labor L, are used to produce two goods, labor-intensive X_1 and capital-intensive X_2, in two countries, with common constant-returns to scale production functions and perfect competition.[1] The home country A is capital-abundant compared with the foreign country B, which represents the rest of the world. Country B is large enough that we can treat country A as a small open economy, with prices determined, under free trade, in B as though it were a closed economy.

As long as those prices are fixed over time, as they would be if large country B either does not grow or grows in a steady state, then they determine the factor prices and cost-minimizing techniques of production that are consistent with diversification. Taking all of that as given, country A's pattern of production and trade depends on its factor endowments relative to those techniques, which form the familiar cone of diversification of the HO model.

If country A starts with a lower capital-labor ratio than either of these techniques – outside the cone – then it will initially specialize completely in the labor-intensive good X_1. As its capital-labor ratio now grows, it will at first remain outside the cone,

[1] I also make the usual assumption that there are no factor intensity reversals.

continuing to specialize and therefore expanding its output and exports of X_1 as it grows. However, when its capital-labor ratio reaches the cone – that is, reaches the technique for producing X_1 under diversification – then specialization ceases and further growth causes it to shift resources into good X_2, a la Rybczynski. Output of good X_1 now falls with further growth, while output of X_2 becomes positive and then rises. If country A's growth continues unabated, then its output of X_1 will fall to zero, causing it first to switch from exporting it to importing it, and output of X_2 will grow until only it is produced. From then on, further growth maintains complete specialization in good X_2, which is therefore exported, and both output and exports rise indefinitely for the remainder of the growth path.

All of this is represented in Figures 1 and 2. Technology, together with the fixed prices from the world market, determine the two curves OZ_1 and OZ_2 in Figure 1, representing the value of output per worker in the two sectors as functions of their capital-labor ratios. As discussed in Deardorff (1974, 1998b), these two curves can be used to determine patterns of specialization, in a manner analogous to the unit value isoquants of the Lerner-Pearce diagram. The straight line that is tangent to both curves at points *A* and *B* determines the capital-labor ratios in the two industries, $\tilde{k}_1$ and $\tilde{k}_2$, that are consistent with producing both goods.[2] The convex hull of these two curves, $OABZ_2$, then gives the country's per capita income, y, as a function of its capital-labor ratio, k.

The value of per capita output of good X_1 is equal to y for k below $\tilde{k}_1$, since X_1 is the only good produced there. Within the cone of diversification – that is, between $\tilde{k}_1$

[2] These are unique since we have assumed the absence of factor intensity reversals.

and $\tilde{k}_2$ – per capita output of X_1 declines linearly to zero, while per capita output of X_2 rises linearly from zero to become total GDP at $\tilde{k}_2$. Thus the curves *OADE* and $OCBZ_2$ measure value of per capita output in the two sectors. Figure 2 shows these as x_1 and x_2 in the top two panels.

To get the pattern of trade from this, we need some assumption about demand. I will follow much of trade theory in assuming homothetic preferences, so that for given prices demand for each good, *i*, is just a constant fraction, α_i, of total income. One of these is shown as the curve $\alpha_1 y$ in Figure 1, which has the same shape as the national income curve, $OABZ_2$. By comparing this demand for good X_1 with supply from *OADE*, the pattern of trade is determined. This is represented in Figure 2 as the per capita net exports of good X_1, t_1. Trade is assumed to be balanced, so that the per capita net export of good X_2 is just the mirror image of this, t_2, also graphed in Figure 2. Assuming, then, that the capital labor ratio of the country grows monotonically over time, the graphs in Figure 2 show the paths of production and trade in both goods that will occur over time with growth.

This constitutes the most straightforward set of implications of the HO model for evolving patterns of specialization, comparative advantage, and trade. To the extent that a country's growth takes it first to, then through, the HO cone of diversification, its comparative advantage will shift from the labor-intensive good to the capital-intensive good, and its pattern of trade will shift accordingly. Of course, it is also possible that a country's growth will stop, in terms of its capital labor ratio, before it leaves the cone or even earlier. For example, if the country in Figure 1 has a steady state below $\tilde{k}_2$, then it will never specialize completely in good X_2, and if its steady state is below k_0 it will never export X_2. That, in turn, depends on its savings behavior compared to that of the world, an issue that is straightforward to explore in the context of Figure 1 but that I will not examine further here.[3] From here on, I will simply assume that a country's growth may take it to all interesting levels and see what they entail. It should be understood that these paths may well be truncated at lower levels for countries that do not save or otherwise grow sufficiently fast and far.

A Growing World

I have assumed so far that the rest of the world remains stationary as our country of interest grows. That is not a very interesting case. But to allow for growth in the rest-of-world I must also specify some detail of how that growth occurs. Is the world too starting out of steady state in an otherwise stationary growth model? Or is technical progress permitting it (and presumably also country A) to grow? If so, what is the nature of that progress? There are too many possibilities here to say anything very general, but in some special but useful benchmark cases, we can say quite a bit. I will consider two: A Cobb-Douglas world that is out of steady state, and a world of constant labor-augmenting technological progress.

[3] For example, suppose that good X_2 is the investment good and used only for that purpose, while growth is caused as in the Solow (1956) growth model by savings proportional to income interacting with population growth and capital depreciation. Country B, the large rest-of-world, will be in steady state with a savings curve similar to $\alpha_1 y$ in Figure 1 crossing a population growth/depreciation ray where it also crosses $OCBZ_2$, since as a large economy its trade must be negligible. Country A, then, with the same structure, will have a steady state to the right (or left) of country B's depending on whether its savings rate is higher (or lower) and/or its population growth/depreciation rate is lower (or higher) than country B's. If A's savings rate is high enough compared to B's, for example, it can have a steady state on the high side of the cone, and its growth can follow the complete path described in the text.

Consider first the Cobb-Douglas world. That is, suppose that both the production functions and the utility function are Cobb-Douglas, with therefore constant fractions of income spent directly on each good and indirectly (since the world is a closed economy) on each factor, γ_L.and γ_K. Then the capital labor ratio of the large rest-of-world, country B, completely and simply determines the prices of factors in country B and the prices of goods in both A and B.[4]

$$\omega^B = \frac{w^B}{r^B} = \frac{\alpha(1-\beta_1)+(1-\alpha)(1-\beta_2)}{\alpha\beta_1+(1-\alpha)\beta_2}\frac{K^B}{L^B} = \frac{\gamma_L}{\gamma_K}k^B \qquad (1)$$

$$p = \frac{p_1}{p_2} = \frac{(1-\beta_2)^{(1-\beta_2)}{\beta_2}^{\beta_2}}{(1-\beta_1)^{(1-\beta_1)}{\beta_1}^{\beta_1}}\left(\frac{\gamma_L}{\gamma_K}\right)^{\beta_2-\beta_1}\left(k^B\right)^{\beta_2-\beta_1} = \frac{\pi_2}{\pi_1}\Gamma^{\beta_2-\beta_1}\left(k^B\right)^{\beta_2-\beta_1} \qquad (2)$$

where α is the share of expenditure on good X_1 and β_i is the capital share in producing good X_i. These prices also determine the bounds of the diversification cone:

$$\tilde{k}_i = \frac{\beta_i}{1-\beta_i}\Gamma k^B \qquad i=1,2 \qquad (3)$$

Notice that both ω^B and $\tilde{k}_i$ are simply proportional to k^B. This suggests that the conditions confronting small country A, coming to it from country B, will also be proportional to k^B, and that it can be analyzed in terms of its own capital-labor ratio relative to k^B.

Indeed, if we define v_i as the value at world prices that an owner of some amounts of capital and labor, K^A and L^A, such as country A can generate per worker by producing good i, this turns out to be

$$v_i = w^B{\pi_i}^{-1}\Gamma^{-\beta_i}\left(\frac{k^A}{k^B}\right)^{\beta_i} \qquad i=1,2 \qquad (4)$$

[4] See the Appendix for derivations of these and subsequent formulas.

That is, suppose that we measure in a graph like Figure 1 the values of output per worker available to country A as a function of its capital-labor ratio relative to the Cobb-Douglas rest-of-world, k^A/k^B. This will look just like the curves in Figure 1, and it will remain stationary even as the rest-of-world grows and k^B changes. Of course, if k^B is in fact growing, then w^B will be growing as well (though not linearly), and country A will be benefiting in real terms even when it is not moving to higher levels of y/w^B.

What this says, then, is that in this special Cobb-Douglas case, everything I said about changing patterns of specialization and trade continues to be true, so long as we interpret k in Figure 1 as the country's capital-labor ratio *relative to* the world: k^A/k^B. Of course, this means that country A can be growing and still have its relative capital abundance declining, if it is growing more slowly than the world. Thus a country may start out specialized in the labor-intensive good and remain there forever in spite of its own growth, if that growth is too slow. And another country could start with a relative abundance of capital and thus specialize in X_2, then grow but more slowly than the world, and as a result move down, not up, the ladder of comparative advantage. Such a country would move over time to the left, not right, along the curves of Figure 2.

A second and even simpler way to allow for growth in the world is to assume that it occurs through a simple form of technological progress. Suppose that labor in country B becomes more productive over time at a constant rate g that is the same in both industries. Then in steady state the world's capital stock will have to grow as fast as its effective labor force in order to keep its effective capital-labor ratio constant. In this steady state, per capita outputs and income will all be growing over time at the rate g.

The story of Figures 1 and 2 will again still apply, but with k now measuring the ratio of capital to effective labor.

If country A shares in the same technological progress as the world, so that the effectiveness of its labor is also growing at rate g, then the same things that might cause it to grow with static technology will cause it to grow relative to the world here. However, it will again be possible, as it was with foreign growth in the Cobb-Douglas case, for a country to grow in the sense of having a rising per capita income but at the same time experience a fall in its ratio of capital to effective labor. In that case it will again move over time towards, or further into, the region of specializing in the labor intensive good.

Figure 3 combines these two cases of foreign growth in what is otherwise the same as Figure 1. The value in each sector of output per effective units of labor, E, is represented as functions v_i of the country's ratio of capital to effective labor in its endowment, compared to the same ratio for the world, $\kappa=(K/E)/(K^W/E^W)$. This relative factor endowment variable can move to the right or left over time, depending both on the country's rate of capital accumulation compared to the world, and also on its rates of technological progress and population growth compared to the world. The cone of specialization is determined within the world market by the ratios of capital per effective labor in each industry to the world's endowment as in (3). If the country shares the same homothetic preferences as the world, then the division between exporting one good and exporting the other is just $\kappa_0=1$, since it will trade based simply on whether it has more or less capital per effective labor than the world.

Both of these cases are indeed special, and in general it will not be true, with growth in the world, that we can analyze one country's structure with a stationary figure like Figure 3. In general, growth in the world will cause prices to change in ways that will shift the curves of Figure 3 and that cannot be neutralized by re-normalizing the variables on the axes. Nonetheless, it seems undoubtedly correct and general that in a growing world the pattern of a country's specialization and trade will depend on the evolution of its own factor endowments and technologies relative to those of the world.

This will continue to be true as we allow for additional complications in the static model, such as additional goods and factors in the following sections of the paper. I will therefore not deal explicitly with foreign growth in those sections, leaving it to the interested reader to adapt their messages to that situation, as I have done here.[5]

III. More Goods and Cones

Suppose now that there are more than two goods, but still only two factors. There are then two kinds of world equilibrium in the HO model, depending on how large are the differences across countries in factor endowments, compared to differences across industries in factor intensities.

If endowments are not too different, then it is possible for the world to replicate the equilibrium of the "integrated world economy" (IWE) that would arise if factors were mobile across countries, even though they are not. In the IWE, countries share the same factor prices. With the same technologies, they are therefore all able to break even producing any and all of the goods. In the corresponding trading equilibrium, therefore,

[5] It is not quite as simple as that sounds, however, once we are in a world of multiple cones. For then, the rest of the world does not behave as a closed economy, as I assumed in developing the Cobb-Douglas case above.

there is factor price equalization (FPE), and while countries may not in fact produce all goods, they could.

The situation is shown in Figure 4, where the curves representing value of output per worker of three industries are all tangent to the same straight line, ABC. These tangencies define a single cone of diversification, bounded by the capital-labor ratios of the least and most capital-intensive goods, $\tilde{k}_1$ and $\tilde{k}_3$. Exactly which goods will be produced, and in what quantities, cannot be known for a single country within this cone, since there are several ways of fully employing the two factors in the three industries, and the world market can make up any difference between supply and demand. Therefore, for example, while we know that the output of the most labor-intensive good will be all of GDP for capital-labor ratios below $\tilde{k}_1$ and zero above $\tilde{k}_3$, that output can take on a range of values within the cone. In fact, depending on whether the country also produces good X_2, good X_3, or a combination, per capita output of X_1 can be as small as the line *AE* or as large as the line AF. There are similar ranges for the other goods, although I have not drawn the lines needed to identify them. In particular, the per capita output of the middle good, X_2, could be as low as zero throughout the cone, and it could be as high as all of GDP at the capital-labor ratio $\tilde{k}_2$. Thus in this single cone equilibrium with multiple goods and FPE in the world market, the HO model does not tell us completely what the pattern of specialization and trade will be as a country grows.[6]

[6] The model does have much firmer implications for the factor content of production and trade, if that is of interest. Indeed, the factor content of production is trivially equal to the country's factor endowments, and therefore becomes more capital intensive with growth of the capital-labor ratio. Likewise and somewhat less trivially, the factor content of trade (under identical homothetic preferences) is given exactly by the difference between its own factor endowments and its expenditure share of world factor endowments. This is the Heckscher-Ohlin-Vanek prediction that has been the basis of most empirical tests of the HO Theorem.

More interesting, in my view, and more plausible for the world we live in, is the alternative HO equilibrium with multiple cones. If factor endowments are not sufficiently similar, in a sense made explicit in Deardorff (1994), then it is impossible to replicate the equilibrium of the IWE within the constraints of the country factor endowments. Prices on the world market will adjust instead to an equilibrium in which different countries or groups of countries can produce different goods, at different sets of factor prices. This can happen in a variety of ways, especially when the number of goods and countries is large, but the simplest case is shown in Figure 5.

Here, prices of three goods are such that no common tangent exists for all three of the curves representing value of output per worker. Instead there are two tangents, one to the curves for X_1 and X_2, and another for X_2 and X_3.[7] These tangencies define cost minimizing techniques of production for the respective goods subject to the factor prices implicit in those tangents, and therefore to be used within the cones where those factor prices prevail. In the lower cone, only goods X_1 and X_2 are produced, using the capital-labor ratios $\tilde{k}_1$ and $\tilde{k}_2'$. In the upper cone, only goods X_2 and X_3 are produced, using the capital-labor ratios $\tilde{k}_2''$ and $\tilde{k}_3$. Good X_2 is produced in both cones, using different techniques in response to their different factor prices. It is also the only good produced between the cones, using all factors available.

Figure 5 describes the world economy, and to be an equilibrium it must be true that different countries have different factor endowments, so that all goods can be produced somewhere. At a minimum, there need to be two countries, one with endowment ratio less than $\tilde{k}_2'$ and the other greater than $\tilde{k}_2''$, so that X_1 and X_3 are produced somewhere. One of these countries may completely specialize (either below $\tilde{k}_1$ or above $\tilde{k}_3$), but both cannot, since X_2 must also be produced.

In general the world could include a great many countries with their endowments distributed along the k axis, displaying every possible pattern of specialization except production of both X_1 and X_3. Prices are of course endogenous, and will have adjusted to the endowments of the countries, not the other way around. If, for example for the prices shown, too many countries' endowments lay between $\tilde{k}_2'$ and $\tilde{k}_2''$, so that too much X_2 was produced, then the price p_2 would fall, pulling the two cones closer together and shifting factors into producing the other goods.

I will simply assume that this problem has been solved, that the world equilibrium is as shown in the figure, and that this world of many countries is very large (and also stationary) compared to the country whose growth I now want to consider. My focus country will therefore take the prices in Figure 5 as given, producing and trading along its growth path as the figure dictates. Once again, the diagonals *AF*, *EB*, and now also *CH* and *GD* tell us the per capita outputs of the various goods as functions of k.

Figure 6 shows the paths of output and trade, much as in Figure 2 except that I have now put all three goods into the same panel. Production is essentially given directly by Figure 5, while trade would be implicit in Figure 5 with the addition of homothetic

---

[7] A third exists for X_1 and X_3, but it is dominated by the other two. Note that, as in the analogous construction in the Lerner-Pearce diagram, a price p_2 low enough for that tangent to dominate could not be an equilibrium, since good X_2 would then not be produced.

tastes to determine demand. In addition, the bottom panel now shows the paths of the wage and the rental price of capital, since their behavior may be of interest.

What we see is that comparative advantage again shifts due to growth, but now not just once. It first shifts from X_1 to X_2 and then again from X_2 to X_3, as the country's capital-labor ratio grows. In effect, the country climbs a ladder of comparative advantage, to more and more capital intensive goods, first producing a good and then, except when it reaches the most capital-intensive good X_3, ceasing to produce it. Patterns of production and trade are therefore both quite variable over time.

In contrast, factor prices are not very variable. The wage rises and the rental on capital falls as capital accumulates, as one would expect, except that both of these changes pause while the country is in a cone of diversification. This is just the well-known factor-price-insensitivity implication of FPE, but it leads in this multi-cone model to factor-price movements that start and stop, then start and stop again.

Many Goods

What happens as we move to allowing more and more goods? In effect what we will see will be a combination of the cases considered already, with the ladder of comparative advantage potentially including more rungs. In fact, however, we do not know how many cones we will get with a larger number of goods. Therefore we do not know what mix we will find of, on the one hand, movements in and out of cones with many ups and downs of output and trade as in Figure 5, and, on the other hand, passage through cones with multiple goods being produced in indeterminate quantities as in Figure 4.

Consider the two extremes that are possible with more than one cone in the case of large numbers of both goods and countries. On the one hand, prices may sort the many goods into just two cones, with most countries in one or the other and able to produce all goods in their respective cones. The picture will then look much like Figure 5, but with many goods tangent to the segment *AB* and many more tangent to *CD*, as in Figure 7a. Within each cone, since there are then more goods in the cone than factors, production and trade is again indeterminate, as in Figure 4. The world is divided into just two groups of countries, perhaps trading to some extent among themselves, but trading primarily with the other group.

In contrast, it is also possible with many goods and countries that prices will adjust to create a great many cones, perhaps as many as there are countries, so that each country resides in its own cone. Such a case is shown in Figure 7b, where essentially the same production functions from Figure 7a have been scaled slightly upward and downward, reflecting slightly different world prices, so that six different cones are formed. Country factor endowments are not shown in these pictures, but for this to be an equilibrium, there must be country endowments in most of the cones.[8]

In the case of one country per cone, every good will be produced in only one, or at most two, countries. Each country will then export what it produces to all, or almost all, others. Note that the bilateral patterns of trade are very different here from the two-cone equilibrium in Figure 7a. In that, countries primarily exported only to the other cone, and only to countries that were either more or less capital abundant than themselves, not both.

---

[8] Strictly speaking, as drawn in Figure 7b, one could get by with countries in every other cone, since all but the extreme goods there are produced in two cones. However, if we think of there being still more goods, with tangencies interior to each of the cones, then each cone must be inhabited by at least one country.

But in the second equilibrium of Figure 7b, with as many cones as countries, all countries export to countries that are both more and less capital abundant than themselves, if such exist. This second equilibrium, incidentally, is one that Davis and Weinstein (1999) have recently begun to take seriously in their empirical examinations of trade. It is also the version of the HO model that most easily gives rise to something like the gravity equation in Deardorff (1998a).

When will one kind of equilibrium or the other arise? I don't think we know, although intuition suggests some tendencies. For example, if countries are tightly grouped in terms of their factor endowments with, say, most poor countries having similar capital labor ratios and most rich countries having higher, but also similar, ratios, then the two-cone equilibrium seems almost inevitable.[9] However, if country endowments are very diverse, especially relative to industry factor intensities, then a large number of cones seems likely. Beyond that, I have yet to find much guidance for sorting out this issue.[10]

IV. More Factors

So far I have assumed only two factors. Yet most would agree that an adequate understanding of international trade requires more. The resolution of the Leontief Paradox was, in part, that we need to consider human capital in addition to physical

---

[9] Although, as for FPE itself in Deardorff (1994), what matters is not just the similarity of factor endowments themselves, but similarity relative to the similarity of factor intensities within the industries that will be produced in the cone. This has been explored by Debaere and Demiroglu (1998).

[10] In Deardorff (1997), I examine a growth model that combines HO trade with the growth dynamics of Galor (1996). In that model, countries endogenously grow into two cones, and I may have left the impression that this makes two cones more likely than many. But that model had only three goods. With many goods, the growth dynamics there may lead to one, two, or many cones, depending on how countries

capital and unskilled labor if we want to understand United States trade, even within manufactures. And for trade in agricultural goods and raw materials, one needs to add land and natural resources of various sorts. Unfortunately, extension of the arguments above to even three factors is difficult, in part because of the constraints of drawing in two dimensions, and in part because the HO model loses some of its ability to make firm predictions in that context. What I will do here is just review two contributions that others have made to understanding trade patterns with three factors, and ask what these contributions tell us about how these change during growth.

The Krueger Model

Krueger (1977) suggested that paths of economic development could be usefully understood with a tractable hybrid of the two-factor, many-good HO model and a specific factors model. In her model, there are three factors: land, labor, and capital. Land and labor are used without capital to produce an agricultural good, while capital and labor are used without land to produce a larger number of manufactured goods. Therefore, land and capital are specific to agriculture and manufacturing respectively, while labor is mobile between them, exactly as in the specific-factors model of Jones (1971). However, within the manufacturing sector, the model behaves like the two-factor, many-good model that we have been considering here, and Krueger was explicit in treating the many-cones version of it. The Krueger model, then, is really a special case of a general 3-factor, many-good HO model, but it is a very interesting and tractable special case,

---

are grouped in their initial conditions. It therefore does not answer the question of how many cones there will be, but only of whether the number of cones is likely to change over time.

thought to be an especially useful approximation of the situation of many developing countries.

In Deardorff (1984) I offered a geometric exposition of Krueger's model, combining the Lerner-Pearce picture of the HO model for the manufacturing sector with the equally familiar specific-factors diagram for allocating labor between agriculture and manufactures. I will not reproduce that analysis here, but I will provide a variant of another diagram that I derived from it. This depicts regions of specialization and diversification in factor space, using the convenience that these depend only on the endowments of land and capital per worker, so that they can fit into two dimensions.

Figure 2 in Deardorff (1984) showed these regions for just three manufactured goods and therefore two cones of specialization. Here, in Figure 8, I extend that figure to include five goods and four cones, in order to stress the model's implied ladder of specialization. The integers in the figure represent which of the five manufactured goods can be produced inside each region of relative factor endowments.

For example, with very little of both capital, K, and land, T, per worker, only the most labor intensive manufactured good, X_1, is produced in addition to the agricultural good (which is produced everywhere in the figure and therefore not mentioned). That may seem obvious, but in fact a country with little capital may not produce the most labor-intensive good if it is better endowed with land. Thus as you move to the right near the T/L axis, you move into regions producing ever more capital intensive goods. The reason is that having more land raises the marginal product of labor and thus the wage that it must be paid. This in turn induces the manufacturing sector to economize on labor by producing more capital intensive goods.

The cones of diversification show up here as the triangles emerging up and to the left from points *t'*, *t''*, etc. along the *T/L* axis. The triangle labeled "1,2", for example, shows all of the factor endowment combinations that permit goods 1 and 2 both to be produced, exactly like the cones in the earlier figures. These triangles are anchored on the vertical *K/L* axis by the same capital-labor ratios discussed earlier, since along that axis, with zero land, there are only two factors. The presence of positive land causes diversification to occur for lower economy-wide capital-labor ratios than these, simply because some labor is being used in agriculture. The ratios employed within the manufacturing industries are the same.

With this picture available, it is a straightforward matter to describe paths of specialization and trade. For example, if a country's labor force and endowment of land were both constant over time and it were to accumulate capital, then it would move straight up in the Figure, crossing in and out of diversification cones exactly as we discussed before. The fact that some labor is being employed with the third factor, land, really does not matter in this case. Likewise, if growth could somehow be achieved by adding to the stocks of both land and capital per worker – a possibility that is perhaps conceivable only here in the Netherlands – then the growth path will angle up and to the right in the Figure. Again the country will climb the ladder of comparative advantage to ever more capital-intensive manufactured goods.

But the most plausible growth path for most countries is neither of these. Instead, since land is normally fixed but the labor force grows with population, we can expect most countries to move to the left in the figure, not to the right. Depending on how rapidly capital also accumulates compared to labor, the growth path may take a country not to more capital-intensive production but to less, during its early stages.

Such an example is the one shown in Figure 8 by the dashed path starting from point *n*. This is a country starting so well endowed with land that its high wage makes it specialize in only the most capital-intensive good, X_5. With so little capital, it may not even produce enough of that to export it, but with a large number of goods available and only one produced, it seems reasonable to suppose that it does export it.

Now we let both the labor force and the capital stock grow over time, with capital growing faster than labor but only by a little. Movement in the figure will then be primarily to the left, as shown, and the country will move down the ladder of comparative advantage in spite of its growth. It will first, and briefly it seems, diversify into X_4 as well as X_5, but then soon switch completely to producing (and presumably exporting) only X_4. As drawn, this pattern of shifting production and trade will be repeated by a shift into X_3 and then into X_2 before it finally turns around and heads back the other way. Only then does it begin to climb the ladder of comparative advantage in the conventional way.

This behavior may seem unusual, but I suspect that it is not. A country may be well-endowed with natural resources and live quite a good life while its population is sparse. It's high wages will then lead it to manufacture, if anything, only goods that do not use too much of its scarce and therefore expensive labor. Over time, however, if population grows, labor can become abundant after all. And even though capital may grow with it apace, the falling wages will at first push labor into more and more labor-intensive occupations. This does not seem an outlandish story to tell about many countries whose labor abundance today arose as much from population growth as from some lack of saving.

The Leamer Model

Leamer (1987) also made the point that adding land as a factor can make a big difference for paths of development. His model imposes no limits on which of three factors can be used in various industries, a flexibility that he made tractable with an ingenious geometric device for displaying both endowments and intensities of three factors in two dimensions. The Leamer triangle puts one factor at each vertex, then measures relative factor quantities by distance from the opposite side of the triangle. Each vertex therefore represents only a single factor, while points interior to the triangle represent combinations of all three factors, in pairwise ratios that are constant along rays from the vertices.

To simplify the HO model, Leamer assumed that factors are used only in fixed proportions, so that industries are represented by single points in the triangle regardless of factor prices. These points then form small triangles inside the larger triangle, and these in turn correspond to cones of diversification. There are no regions of complete specialization separating these cones, as has been the case here with factor substitution. The points alone do not fully determine the cones of diversification, however, since the small triangles can be drawn in various ways for a given set of points. It therefore requires prices to determine how they all fit together.

To illustrate, Figure 9 shows a Leamer triangle for the three factors unskilled labor, skilled labor, and capital. Leamer dismissed this combination as uninteresting, on the grounds that physical and human capital tend to accumulate together, and I certainly agree that for many purposes adding land, instead of human capital or skilled labor, is more informative. But having already done that above, I would prefer to focus next on how trade may depend on the nature of a country's growth, recognizing that countries do in fact influence, through their policies and cultures, the choice between human and physical capital accumulation.

Following Leamer in other respects, however, I arbitrarily select several industries with different factor intensities to describe the world. These are based on nothing more than a very impressionistic idea of actual technologies, plus a desire to keep things fairly simple and to have each industry name start with a different letter of the alphabet:

- H: Handicrafts Unskilled labor only
- T: Textiles Much unskilled labor and some capital, no skilled labor
- A: Autos Much capital and some unskilled labor, no skilled labor
- C: Chemicals All three factors
- E: Education Skilled labor and capital only, same proportions as Chemicals
- P: Programming Much skilled labor and some unskilled labor, no capital

The points in Figure 9 embody these assumptions about the factors needed in each industry. The solid lines arbitrarily connecting some of them then delineate, as triangles, five cones of diversification. Countries that happen to be located inside these triangles can produce a mixture of the three goods defining them. For example, a country that has mostly unskilled labor, with only a little capital and skilled labor, will have its endowment close to the *U* vertex of the large triangle, and it will therefore be in the cone HTP, producing a mix of handicrafts, textiles, and programming.

In order to grow, such a country will accumulate capital and/or skilled labor. Depending on the proportions of these two factors that it happens (or decides) to acquire, it may follow either of the two dashed arrows G_1 and G_2 shown in the figure, or of course any of infinitely many others. These paths both keep the ratio of capital to skilled labor constant, at a high level along G_1 and at a low level along G_2, but that is of course no more necessary than anything else I have drawn here.

Along both of these growth paths, the countries start with the mixture of H, T and P that I said above, although the country with more capital produces more T and the other more P. Along both paths also, as drawn, the countries soon pass into the cone TCP, where they cease producing handicrafts and start producing chemicals. But continued growth along G_1 leads soon into cone TCA, where programming is exchanged for producing autos, while growth along G_2 leads eventually into cone PCE where textiles is abandoned in favor of education. The G_1 path also leads even further into cone CAE, where it too begins producing education instead of textiles. In the Leamer model, all of these paths for output are linear in the inputs, and I could draw graphs much like Figure 6 to illustrate them, as indeed Leamer does himself. However, I want to make a different point here.

First, although Leamer does not stress it, the small triangles representing cones of diversification in his pictures could just as easily include additional industries in their interiors that can operate together with those at the corners. That is, just as in Figure 7a, prices may well adjust so that a whole batch of goods, not just three, can be produced within a cone at the same factor prices. Indeed, if there are many more goods than countries, that will be necessary. Therefore, rather than going through the exercise above of trying to identify the major industries and their factor intensities within the Leamer triangle, we should probably better think of the triangle as being sprinkled liberally with dots representing industries with all manner of factor intensities.

What then will define the cones? Again, as in the two-factor model, I think that we do not really know, although it will surely be some interaction between country factor endowments and this distribution of factor intensities. That would have to be the case if there were only a few countries, together with many goods.

As an example, let us construct a Leamer triangle for a world of just three comparably sized countries and many goods. Assume that the three countries have sufficiently different factor endowments to prevent FPE, even between any two of them, so that instead we have three sets of factor prices.[11] I will take country A to be abundant in unskilled labor with therefore a low unskilled wage, country B to be abundant in capital with a low rental, and country C to be abundant in skilled labor, with a low skilled wage. Figure 10 shows iso-factor-cost planes for each of the countries, given these factor prices. For all of them to compete under free trade, each plane must lie further from the origin than the others for some set of factor requirements, so that there exist goods that each country can produce and export to the others. Goods vectors are not shown in Figure 10, but one should think of the space as being well populated with them, so that each country has plenty that it can produce. For this to be an equilibrium, factor prices and the resulting goods prices must also have adjusted so that supply can equal demand for every good and so that each country's trade is balanced. I assume all that to be the case.

[11] The construction here is analogous to that given for two factors in Deardorff (1979).

The resulting Leamer triangle is shown in Figure 11. Notice that the factor planes in Figure 10 divide the Leamer triangle into three regions, one for each country, each of which must contain the factor endowment vector of the country. The latter are shown in Figure 11as the circles labeled *A*, *B*, and *C*. The three regions are not, however, triangles, as Leamer typically draws them, but rather quadrilaterals as they appeared in Figure 10. Nor are the borders of these regions necessarily defined by industry factor requirement vectors, which may or may not have happened to coincide with the intersections of the factor price planes in Figure 10. I have drawn black dots representing a rather large number of industries into Figure 11, more or less randomly, and only a couple of them coincide with the boundaries of regions.

What are the cones of diversification? At first I thought that they were these entire quadrilateral regions taken from the factor price planes, but they are not. Diversification requires production of the goods within the cone, and therefore can only occur with a convex linear combination of their factor requirements. It is critical, for example, that the factor endowment circles lie among, not outside of, the spaces spanned by the factor requirements in the respective cones. These are drawn in Figure 11 as the many-sided dashed polygons within each of the three quadrilaterals formed by the factor-price planes

Consider now the situation that confronts any small country that we add to this world of three large countries. If the small country can fully employ its factors producing two or more[12] of the goods produced by one of the large countries, then it will do so, and it will share that large country's factor prices. But if its factor endowments place it outside all of the three polygons representing the convex combinations of individual large-country industry factor requirements, then it will have to produce goods from two or more of the large countries in order to fully employ its factors. Just which two, as in Leamer, will depend on prices in ways that we cannot know without more information

In Figure 11, therefore, the three major cones of diversification that correspond to the large countries' factor prices are the aforementioned polygons, which are the convex hulls of the dots showing the goods that they produce. I have filled part of the remaining space with somewhat arbitrarily selected small triangles to show what happens outside these hulls.

Now consider again the two growth paths, G_1 and G_2, that might be followed by small countries as they grow within this world of three large and stationary countries. Both start within the diversification cone of Country A – the dashed polygon that encloses Country A's endowment. Both therefore share Country *A*'s factor prices initially, including its low wage of unskilled labor. As they grow, both will at first continue to produce only goods that Country A produces. However, the exact mix of these goods will change, shifting towards the more capital-intensive ones along G_1 and towards the more skill-intensive ones along G_2. Factor prices will not change as long as the small countries remain in A's cone.

At points δ_1 and δ_2, however, both leave the cone and must produce something else. Along G_1, the small country begins to produce good *f* from *B*'s cone together with goods *a* and *b* from *A*'s. These three goods form a little cone of diversification, the small triangle *abf*, for the small country alone. The country soon leaves this cone too, however,

[12] Three are enough, since there are three factors.

passing into *afe*, where it produces two of *B*'s goods, *f* and *e*, together with good *a* from country *A*. Eventually it grows into Country *B*'s diversification cone, shown as another polygon, and from there on its combination of goods is all from country *B* and is indeterminate.[13]

The path G_2 is similar, except that by accumulating more skilled labor instead of capital it heads toward large country *C*'s cone of diversification, instead of *B*'s. Interestingly along the way, however, while it is between large country cones, it briefly produces one of the goods from country *B*: good *g*. It just happens to be the case as I have drawn them, that Leamer's little triangles of diversification that lie along the growth path G_2 include two – *cig* and *ihg* – that extend into country *B*'s factor price plane and include one of its goods.

[13] Were it to continue to grow out the other side of *B*'s cone, it would first produce a combination of goods from *B* and *C*, then pass beyond the convex hull of *all* production points. The return to capital and/or the (excess) wage paid to skilled labor would go to zero, and then, if not before, accumulation would stop.

V. Conclusion

There is nothing new here. We were taught years ago by Heckscher and Ohlin that patterns of production and trade should depend on factor endowments, and all I have done is to illustrate that dependence during the process of factor accumulation that occurs during growth. Nor have any of these illustrations been new, really, since I believe that all of them have appeared before within the literature, or at least in the mind, of trade theory. The point has merely been to bring these pictures together into one place, so that we can refer to them, and so that we can agree on what the process of factor accumulation entails for patterns of production and trade.

That lesson is simply this: As a country accumulates factors of production during the process of growth, it is very likely that the goods in which it has a comparative advantage will change over time, and then that they will change again, and yet again. What matters is not a country's absolute endowments, but its factors compared to those of the world. If it accumulates a productive factor, such as capital, more rapidly than that factor is expanding in the world, then it will eventually move over time up a ladder of comparative advantage based on the abundance of that factor. As it ascends the ladder, it will pass from one cone of diversification to another, shifting production first into and then out of one or more of the goods that can be produced efficiently within each cone. There are different ladders for different productive factors such as capital and skilled labor, and which ladder a country will climb depends on the proportions with which it accumulates them. In addition, since some productive factors such as land cannot be accumulated, it is also possible that movement will be down, not up, one of these ladders of comparative advantage, in spite of the fact that an economy's overall income may be growing.

The importance of all this for growth and development has been known for some time – at least since Krueger (1977) – but it bears repeating. The process of development in the HO model is not smooth. Instead, it requires that industries rise and fall, that resources move into and out of activities that become profitable and then cease to be so. This is bound to be costly for those who must bear the burden of that adjustment.

Appendix

Cobb-Douglas World

Equation (1) follows immediately from ratio of the factor shares:

$$\frac{w^B L^B / E}{r^B K^B / E} = \frac{\alpha(1-\beta_1) + (1-\alpha)(1-\beta_2)}{\alpha\beta_1 + (1-\alpha)\beta_2} . \qquad \text{(A1)}$$

Equation (3) follows from equating factor prices to the ratio of factor marginal products

$$\frac{r^B / p_i}{w^B / p_i} = \frac{\beta_i \tilde{k}_i^{\beta_i - 1}}{(1-\beta_i)\tilde{k}_i^{\beta_i}} , \qquad \text{(A2)}$$

from which, using (1),

$$\tilde{k}_i = \frac{\beta_i}{1-\beta_i} \frac{w^B}{r^B} = \frac{\beta_i}{1-\beta_i} \Gamma k^B . \qquad \text{(A3)}$$

Price can be found from marginal cost as

$$p_i = \frac{w^B}{(1-\beta_i) k_i^{\beta_i}} = \frac{w^B}{(1-\beta_i)\left(\frac{\beta_i}{1-\beta_i}\Gamma k^B\right)^{\beta_i}} = \frac{w^B \Gamma^{-\beta_i} \left(k^B\right)^{-\beta_i}}{(1-\beta_i)^{(1-\beta_i)} \beta_i^{\beta_i}} \qquad \text{(A4)}$$

$$= w^B \pi_i^{-1} \Gamma^{-\beta_i} \left(k^B\right)^{-\beta_i}$$

from which (2) follows immediately. (4) is then

$$v_i = p_i \left(k^A\right)^{\beta_i} = w^B \pi_i^{-1} \Gamma^{-\beta_i} \left(\frac{k^A}{k^B}\right)^{\beta_i} . \qquad \text{(A5)}$$

References

Davis, Donald R. and David E. Weinstein 1999 "Trade in a Non-Integrated World: Insights from a Factor Content Study," in process, (April 29).

Deardorff, Alan V. 1974 "A Geometry of Growth and Trade," *Canadian Journal of Economics* 7, (May), pp. 295-306.

Deardorff, Alan V. 1979 "Weak Links in the Chain of Comparative Advantage," *Journal of International Economics* 9, (May), pp. 197-209.

Deardorff, Alan V. 1984 "An Exposition and Exploration of Krueger's Trade Model," *Canadian Journal of Economics* 17, (November), pp. 731-746.

Deardorff, Alan V. 1994 "The Possibility of Factor Price Equalization, Revisited," *Journal of International Economics* 36, pp. 167-175.

Deardorff, Alan V. 1997 "Rich and Poor Countries in Neoclassical Trade and Growth," Research Seminar in International Economics Discussion Paper No. 402, University of Michigan, June 29.

Deardorff, Alan V. 1998a "Determinants of Bilateral Trade: Does Gravity Work in a Neoclassical World?" in Frankel, Jeffrey, ed., *The Regionalization of the World Economy*, Chicago: University of Chicago Press.

Deardorff, Alan V. 1998b "Does Growth Encourage Factor Price Equalization," Discussion Paper No. 431, Research Seminar in International Economics, University of Michigan, August 10.

Debaere, Peter and Ufuk Demiroglu 1998 "On the Similarity of Country Endowments and Factor Price Equalization," in Peter Debaere, *The Factor Proportions Theory of International Trade and Finance*, Ph.D. thesis, University of Michigan, Chapter 2.

Galor, Oded 1996 "Convergence? Inferences from Theoretical Models," *Economic Journal* 106 (July), pp. 1056-1069.

Jones, Ronald W. 1971 "A Three-Factor Model in Theory, Trade, and History," in J.N. Bhagwati, R.W. Jones, R.A. Mundell, and J. Vanek, eds., *Trade, Balance of Payments, and Growth: Essays in Honor of Charles P. Kindleberger*, Amsterdam: North-Holland.

Krueger, Anne O. 1977 *Growth, Distortions, and Patterns of Trade Among Countries*, Princeton Studies in International Finance, No. 40, (Princeton).

Leamer, Edward E. 1987 "Paths of Development in the Three-Factor, *n*-Good General Equilibrium Model," *Journal of Political Economy* 95, pp. 961-999.

Solow, Robert 1956 "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics* 70, pp. 65-94.

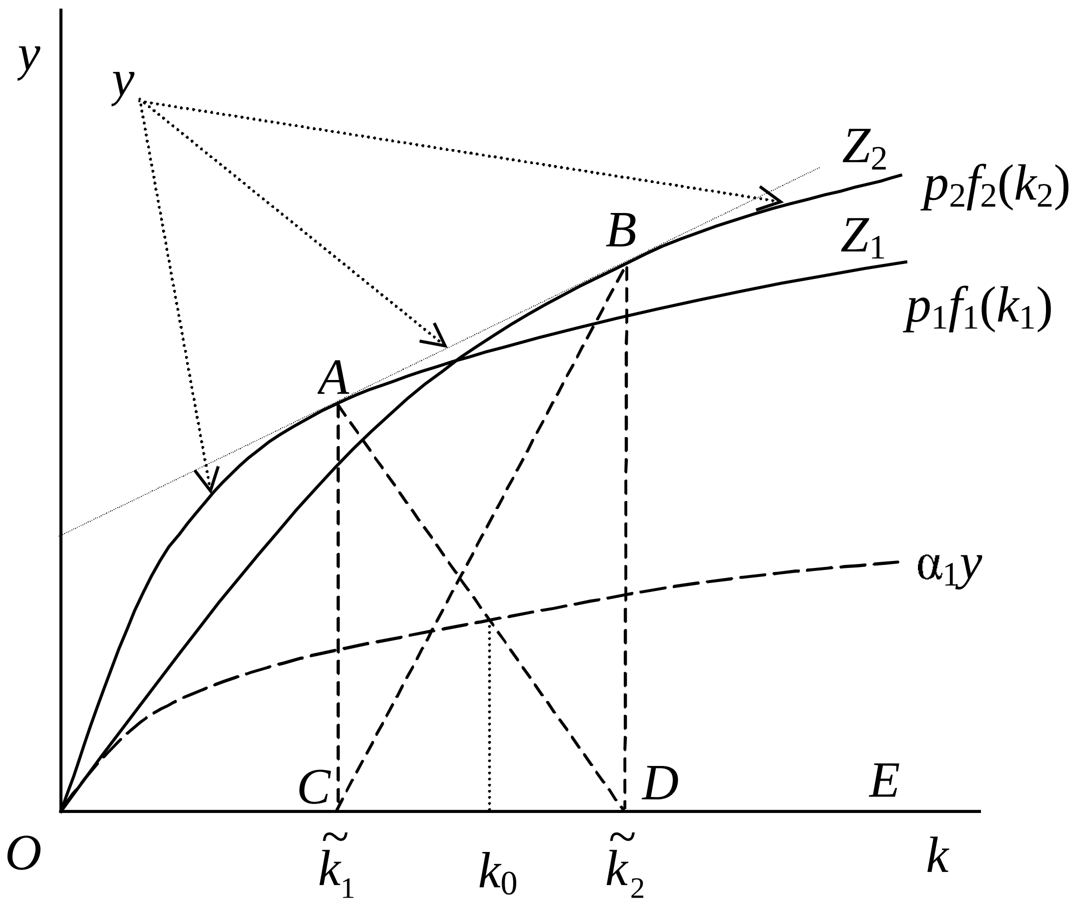

Figure 1
Determination of Specialization
Two Factors, Two Goods

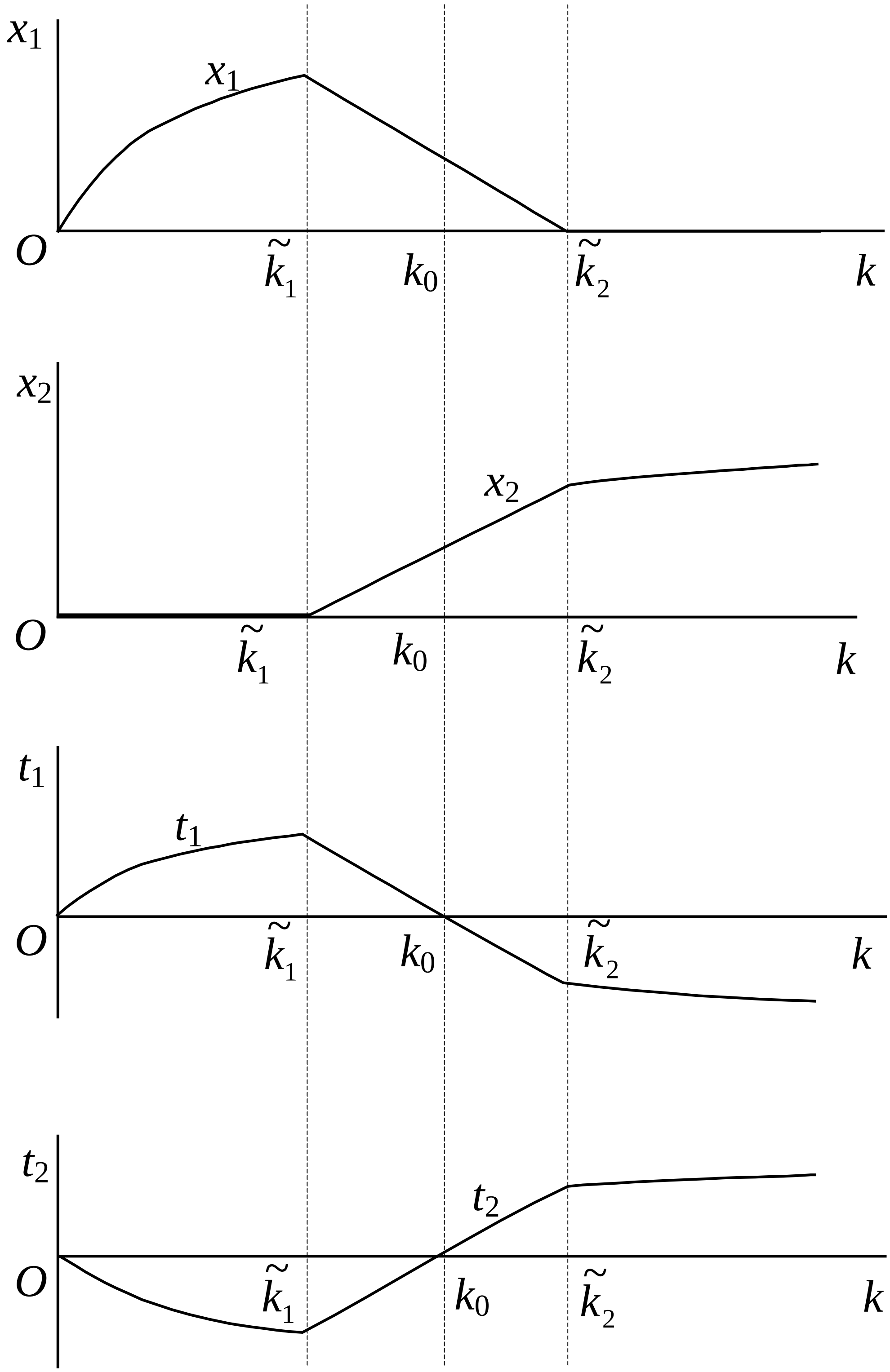

Figure 2
Paths of Output and Trade
Two Factors, Two Goods

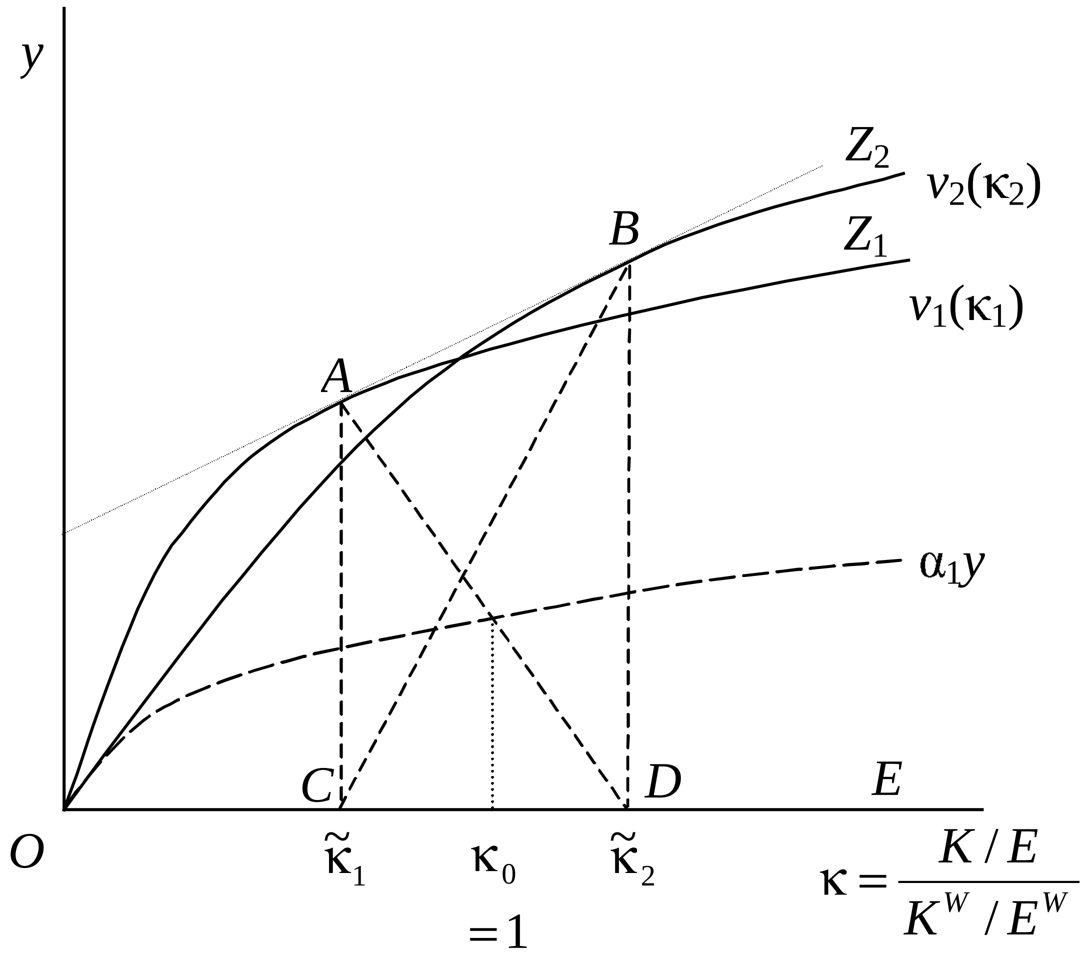

Figure 3
Determination of Specialization
In a Growing World

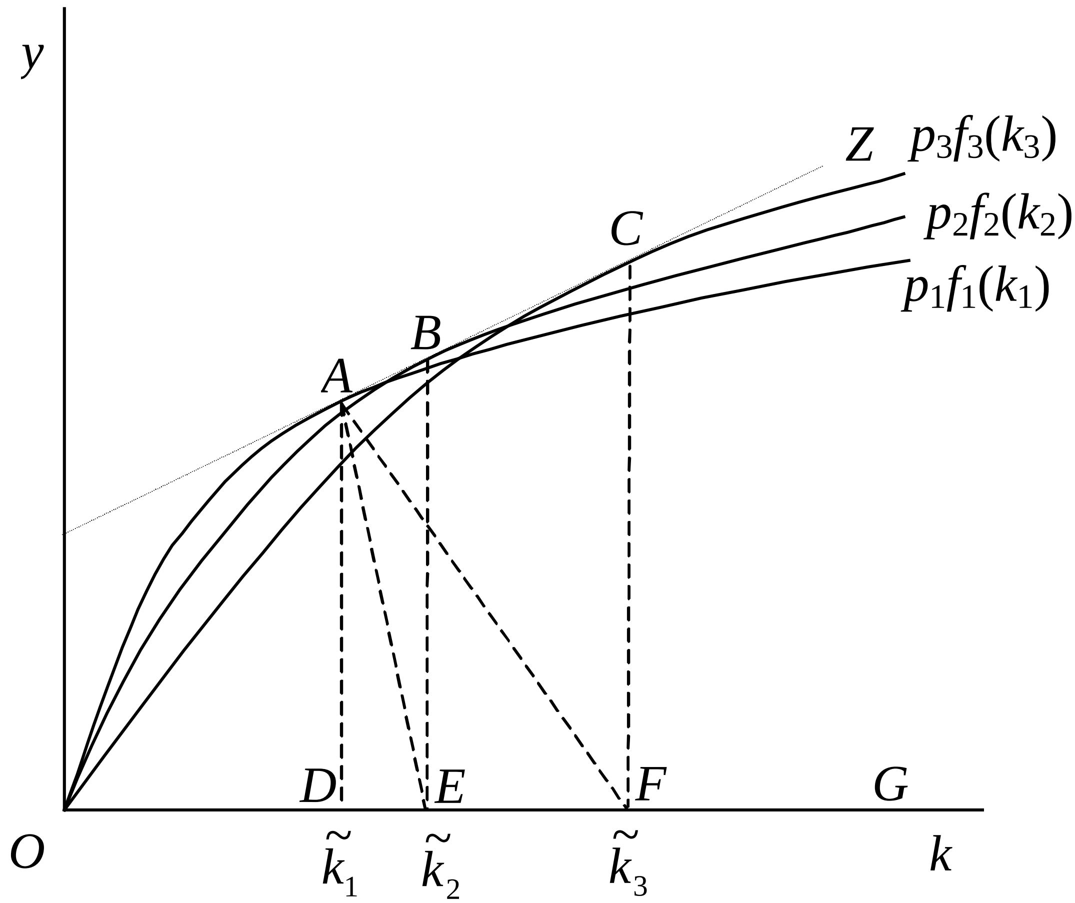

Figure 4
Diversification with
Two Factors, Three Goods

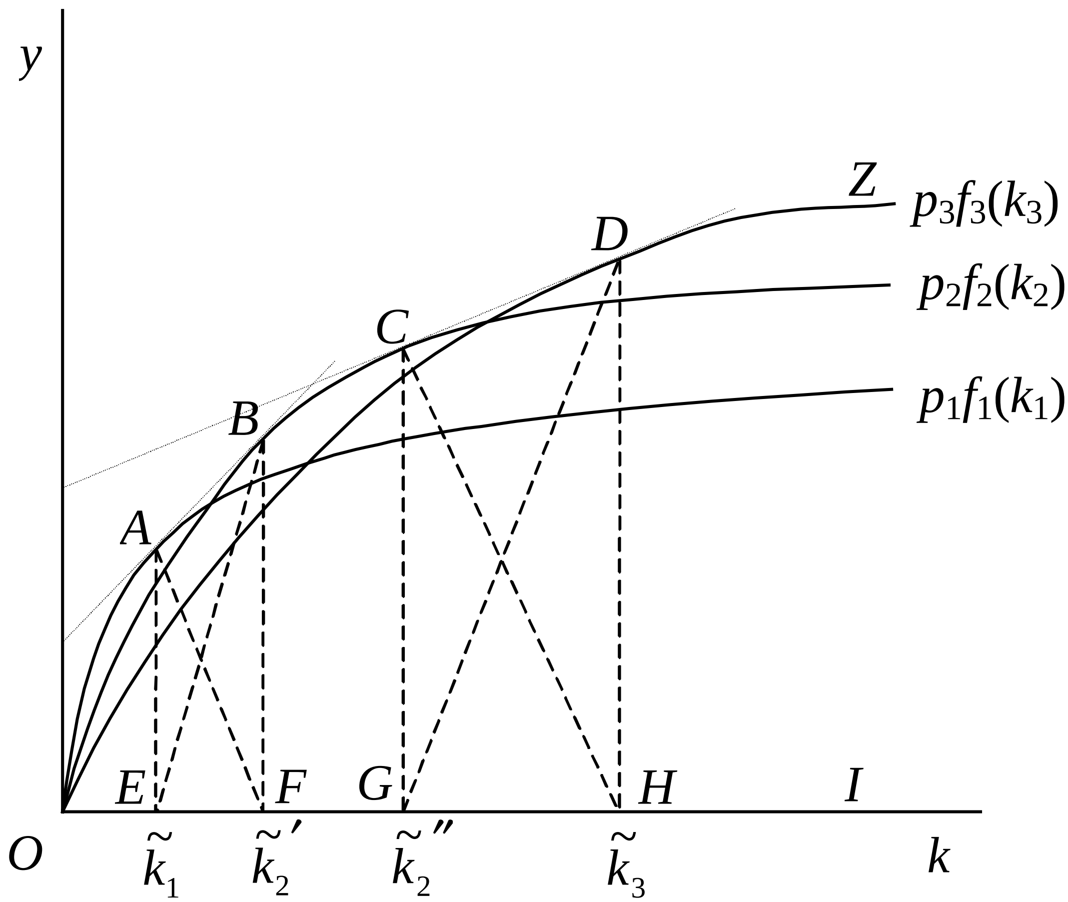

Figure 5
Two Factors, Three Goods,
Two Cones

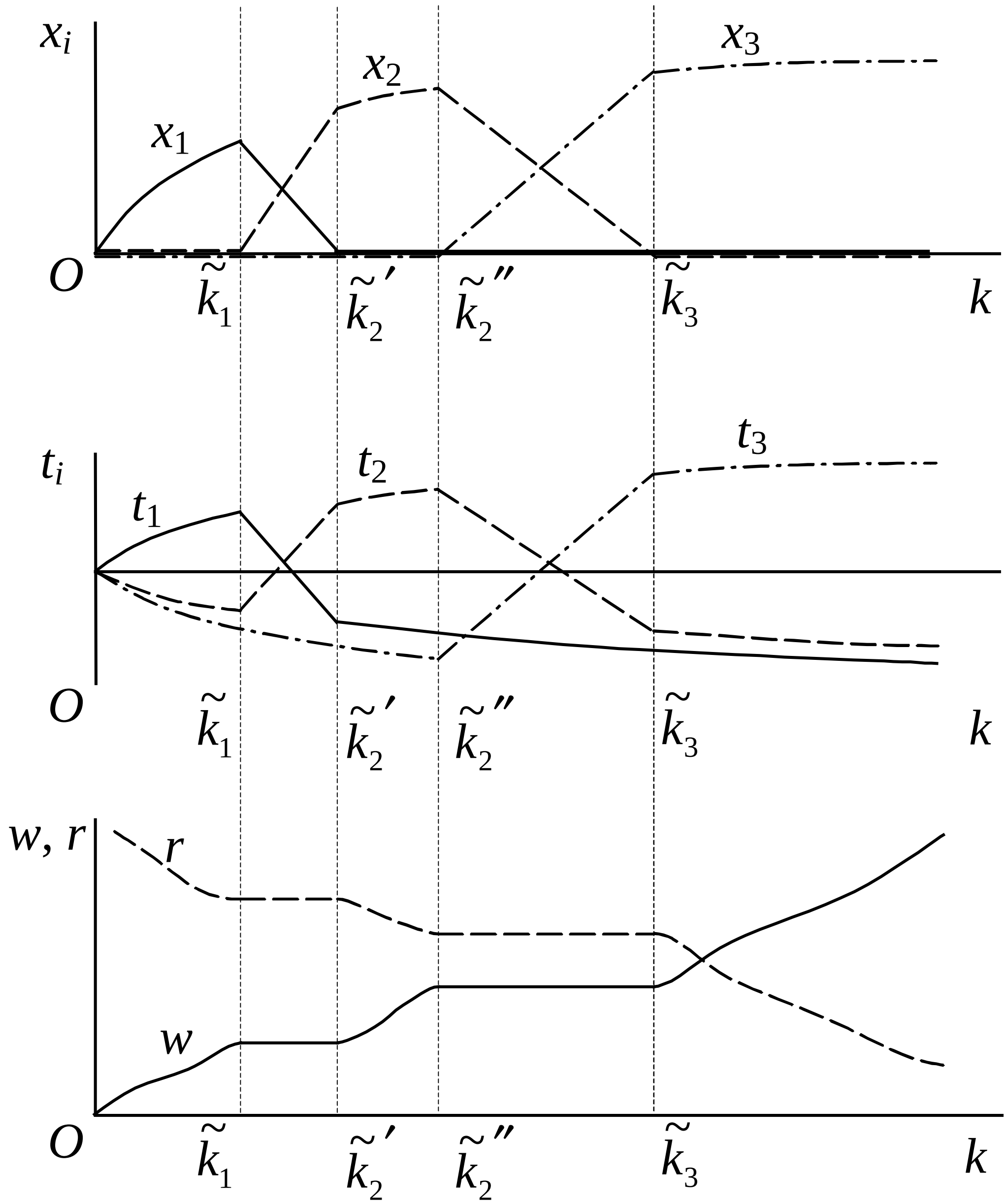

Figure 6
Paths of Output, Trade and Factor Prices, Two Factors, Three Goods

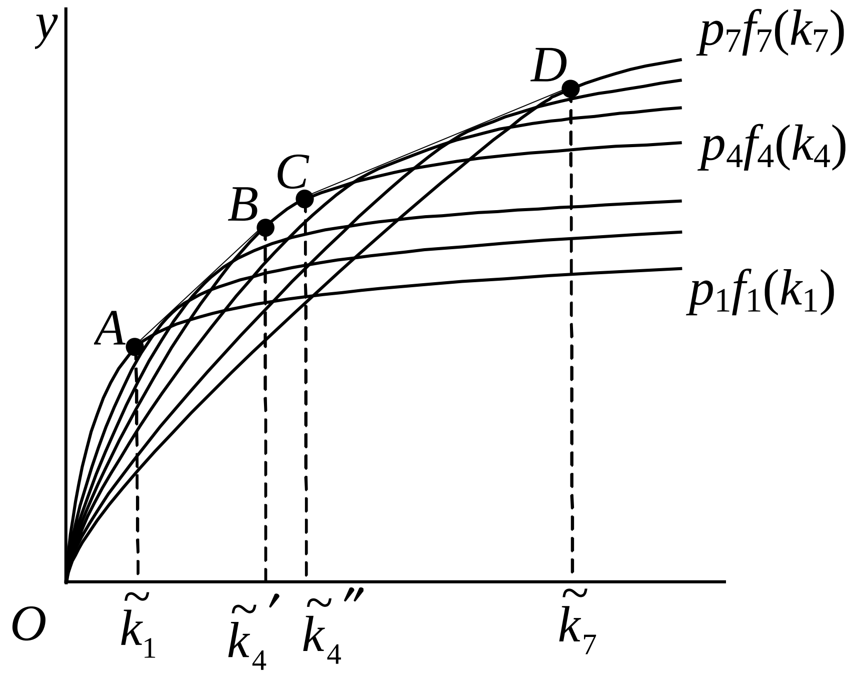

Figure 7a
Two Factors, Many Goods,
Two Cones

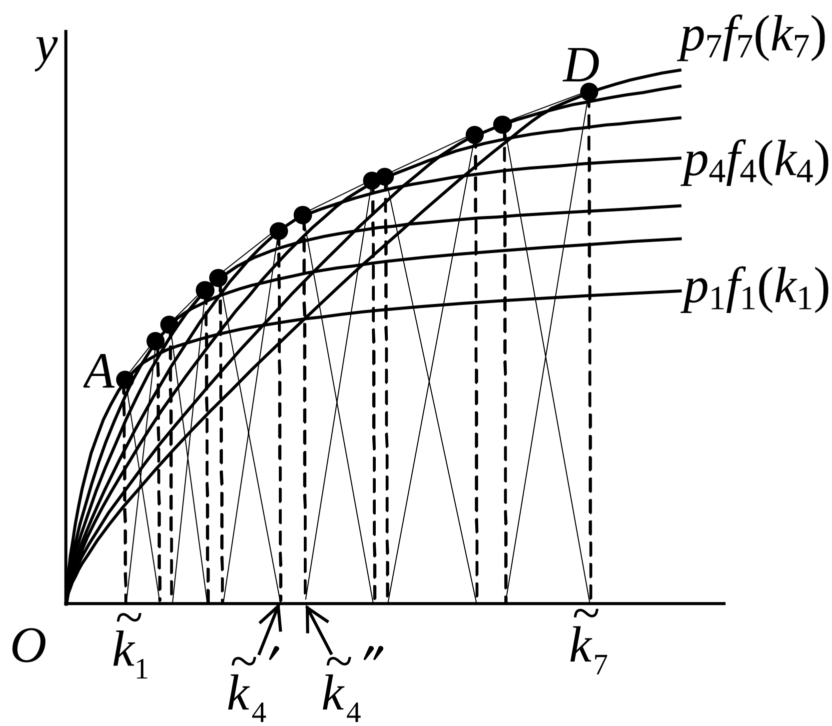

Figure 7b
Two Factors, Many Goods,
Many Cones

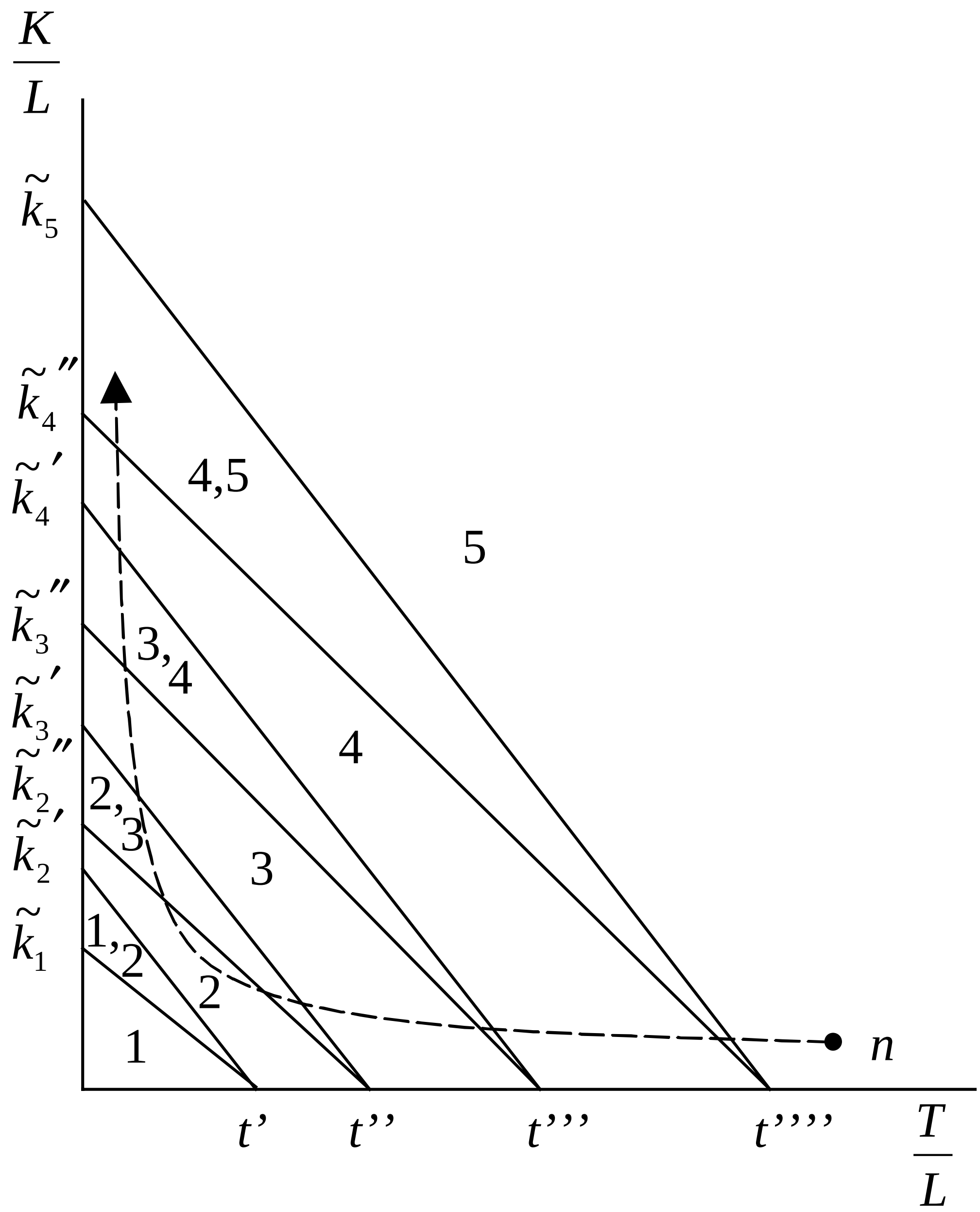

Figure 8
The Krueger Model:
Three Factors, Many Goods,
Many Cones

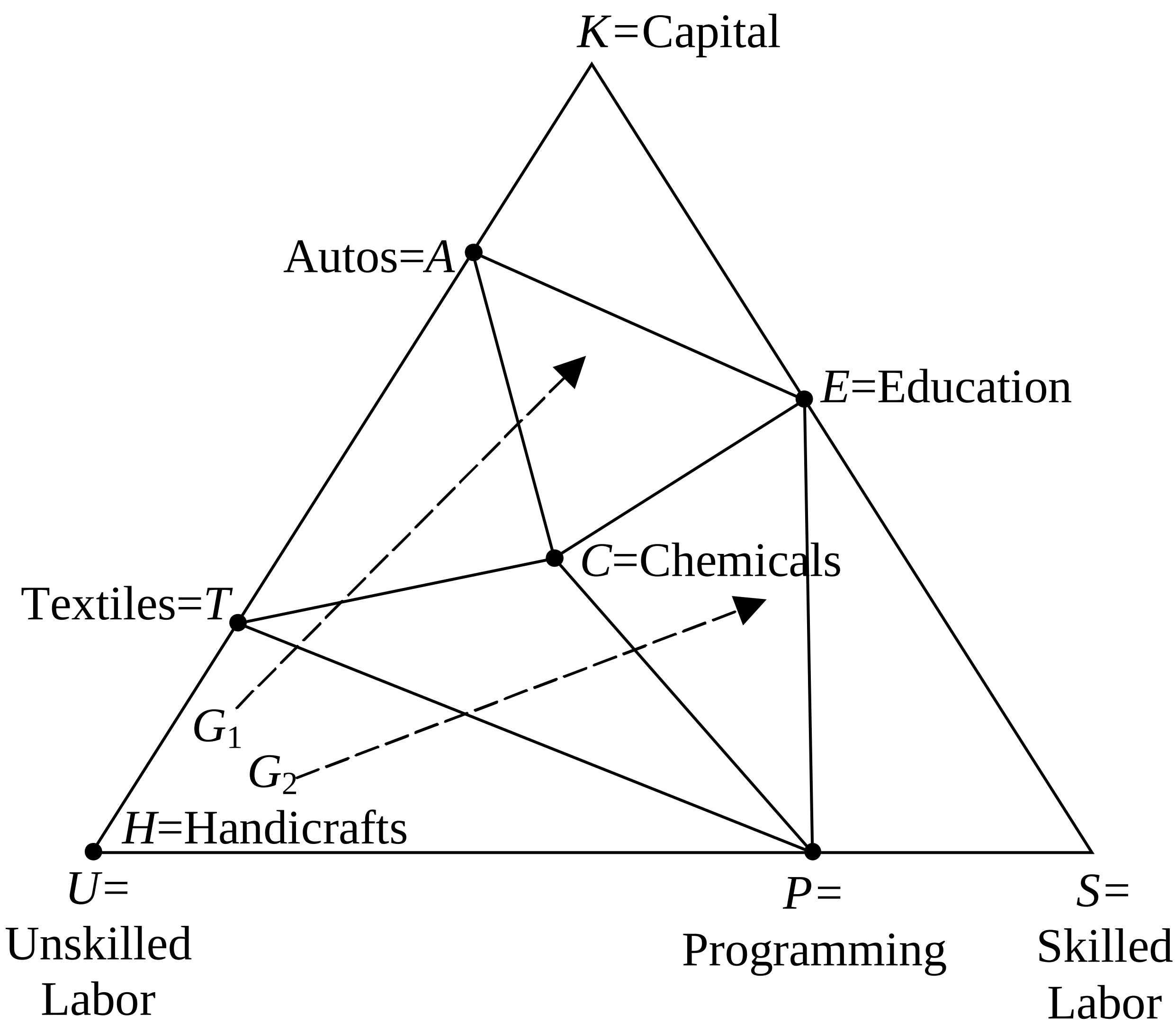

Figure 9
The Leamer Triangle:
Three Factors, Many Goods,
Many Cones

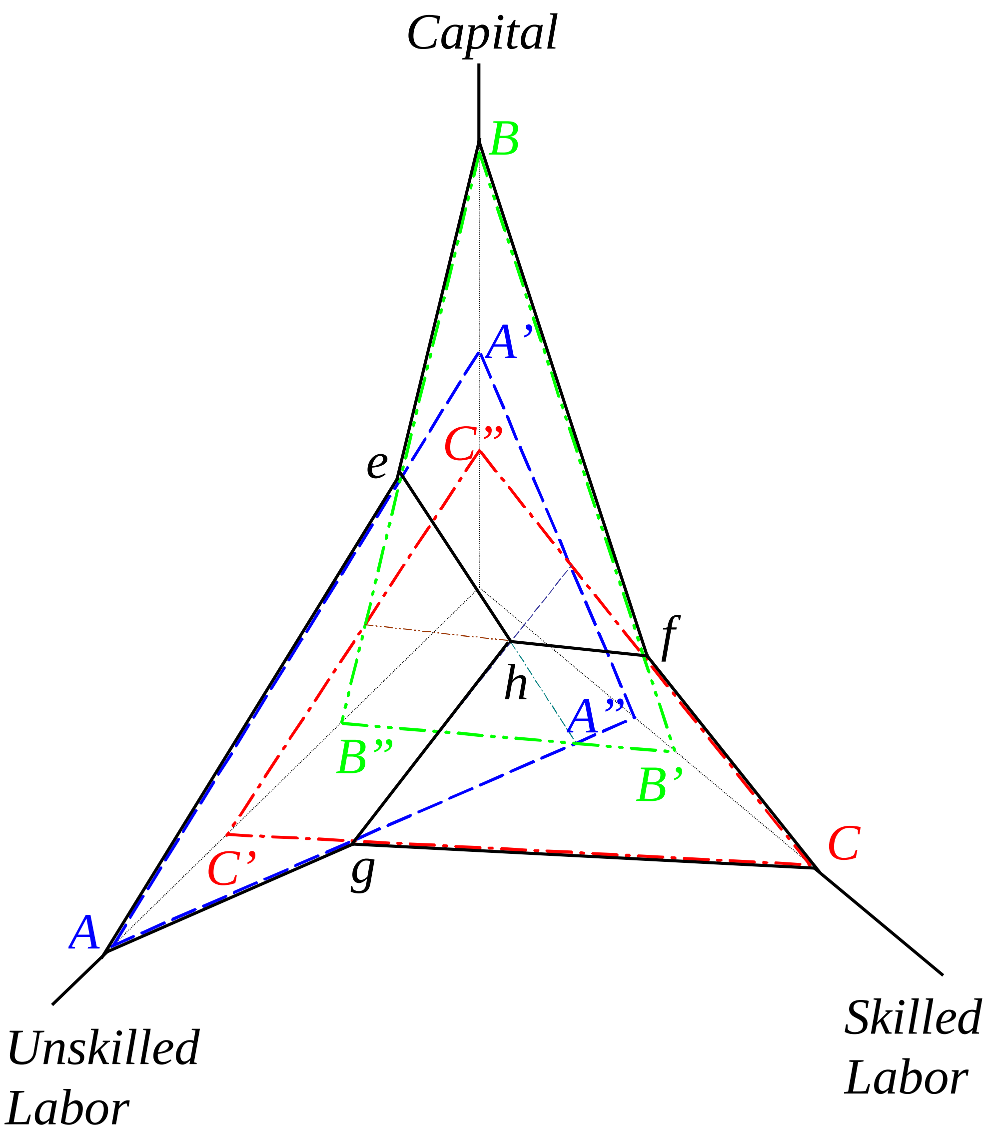

Figure 10
Subdividing Factor Space
into Three Cones

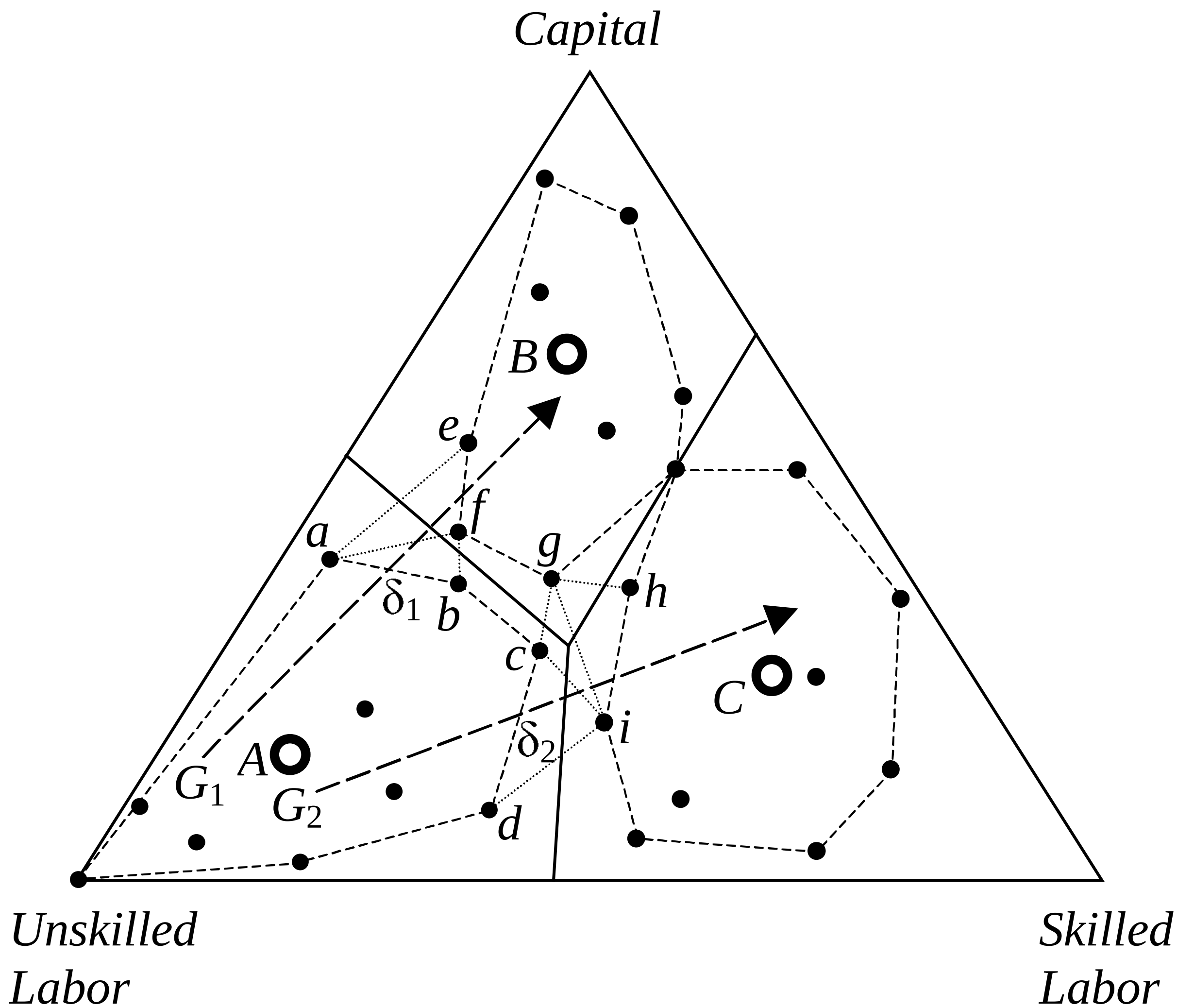

Figure 11
The Leamer Triangle:
Three Factors, Three Countries,
Many Goods