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Duration of Sovereign Debt Renegotiation

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Abstract

The structure of sovereign debt has evolved over time from illiquid bank loans toward liquid bonds that are traded on the secondary market in the past two decades. This change in the debt structure is accompanied with a reduction in the duration of sovereign debt renegotiation; it takes on average 9 years to restructure bank loans, but only 1 year to restructure bonds. In this work, we argue that the secondary market plays an important role — information revelation — in reducing the renegotiation length. We construct a dynamic bargaining game between the government and the creditors with private information on the creditors' reservation value. The government uses costly delays as a screening device for the creditors' type, and so the delays arise in equilibrium. Moreover, the more severe is the private information, the longer the delays are. When we introduce the secondary market, the equilibrium delays are greatly reduced. This is because the secondary market price conveys information about the creditors' reservation and lessens the information friction. We also find that bond financing is more friendly to the debtor country; it increases ex-ante borrowing and investment and ex-post renegotiation welfare of the government.

JEL: F02, F34, F51
Keyword: sovereign debt renegotiation, secondary bond markets, dynamic bargaining, incomplete information

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1 Introduction

The structure of sovereign debt of developing countries has evolved over time from illiquid bank loans to liquid bonds. Before 1990, sovereign countries borrow mainly from commercial banks in the form of syndicated bank loans, which are often customized to the government needs and rarely traded. After 1990, sovereign countries start to borrow mainly in the form of sovereign bonds, which are highly standardized and liquid on the secondary market. This change in the sovereign debt structure is accompanied with a reduction in the renegotiation length. Restructuring of bank loans is prolonged, taking on average 9 years. In early 1990s, even longer renegotiations were expected when governments start to borrow in terms of bonds from a large number of diffused creditors. In reality, however, it takes only 1 year on average to restructure bonds defaulted upon after 1990.

It is of policy relevance to understand why bond debt has shorter delays than bank debt. Delays are inefficient: the government suffers from losing access to international financial markets and the creditors cannot realize investment gains. It has been widely argued that faster debt restructuring would have helped major borrowing countries recover from crisis and restore the momentum of growth at an earlier stage. It is also theoretically relevant to understand ex-post renegotiation outcomes of bank debt and bond debt. Different renegotiation outcomes have direct implications on ex-ante borrowing and default incentives of the government for bank debt and bond debt.

In this paper, we argue that the secondary market plays an important role — information revelation — in reducing the renegotiation length. One important reason for inefficient equilibrium delays is private information. When the creditors' reservation value is private information, the government uses costly delays as a screening device for the creditors' type, and delays optimally arise in equilibrium. Moreover, the more severe is the private information, the longer the delays are. When we introduce the secondary market, the secondary market price conveys information about the creditors' reservation and lessens the information friction. As a result, the equilibrium delays are greatly reduced.

We model the renegotiation of illiquid bank loans by adopting the dynamic bargaining game with private information in Fudenberg et al. (1985). A government defaults on its debt and negotiates with creditors over a new debt contract. Both parties discount the future at the same rate. During the renegotiation, the government suffers a loss in output, and the creditors can seize a fraction of the output loss, which forms their reservation value. The reservation value is private information of the creditors, and the government is informed only about its distribution. In each renegotiation period, the government makes a debt restructuring proposal, and the creditors either accept or reject the proposal. If the creditors accept, the government repays the creditors the proposed offer and avoids the output loss. Otherwise, the renegotiation continues to the next period.

Private information is key to generating delays in equilibrium. Without private information, the renegotiation has no delay: the government proposes the reservation value and the creditors accept immediately. Thus, the government captures all the surplus. With private information, the government needs to propose the highest possible reservation to avoid delays. In this case, the government obtains the least surplus possible. Thus, delays might increase his surplus. A lower offer reduces the probability of acceptance, but increases the surplus when the creditors in fact have low reservation and accept the offer. Thus, the costly delays arise in equilibrium as a screening device of the creditors' reservation.

In the unique perfect Bayesian equilibrium, the renegotiation always ends in finite periods. The key assumption is that the creditors cannot seize all the output loss. This implies that the government obtains a positive sure payoff if he proposes the highest possible reservation to end the renegotiation immediately. As time goes by, the potential surplus from further screening the creditors becomes insignificant relative to the sure payoff, and so the government wants to end the renegotiation immediately. More importantly, when the information friction is more severe, the gains from delays are higher and the maximum renegotiation length are longer.

To model the renegotiation of liquid bonds, we introduce into the basic framework the secondary market trading, following Grossman and Stiglitz (1976). The government defaults on bonds, which are equally held by all creditors. The distribution of the creditors' reservation value is public information. Before the secondary market opens, each creditor receives a signal about their common reservation value. According to their signals, the creditors decide whether to buy one additional unit of bond, to sell or to hold their bond on the secondary market. Some random fraction of the creditors turn out to be noisy trader and sell regardless of their signals. The market price is public information. After the secondary market trading, the creditors with bonds observe their reservation value, and the renegotiation process starts and is the same as before.

We find that the renegotiation length is much shorter with the secondary market. This is because the secondary market price conveys information about the creditors' reservation value. The price is informative because it aggregates the signals that the creditors receive based on the true reservation and according to which they trade. The government updates his belief about the distribution of the reservation value of the creditors using the market price. Thus, the government has more precise information about the reservation value, which reduces the delays in renegotiation.

We also analyze the impacts of the secondary markets on the renegotiation welfare of the creditors and the government and on ex-ante borrowing and investment incentives of the government. The reduction in the renegotiation length increases the total surplus. At the same time, the government gains more bargaining power and derives higher payoffs from the renegotiation. Thus, the renegotiation outcomes of bond financing is more friendly to the debtor country than to the creditors. Furthermore, the delays in the ex-post renegotiation reduce the ex-ante borrowing and investment incentives of the government. Thus, bond financing also improves efficiency in borrowing and investment through its reduction in the renegotiation length.

Our work relates to a large theoretical literature on sovereign debt renegotiation. Bulow and Rogoff (1989), Fernandez and Rosenthal (1990), Kletzer (2003), and Yue (2006), assumes complete information and generates no delays in reaching agreements. Two recent works, Benjamin and Wright (2008) and Bi (2008), analyze the impact of uncertainty on the renegotiation length.[1] Complimentary to their studies, our work focuses on the role of information frictions. Pitchford and Wright (2007) examine delays arising from "free riding" and "hold out" among the creditors. Haldane et al. (2005) show that to a large extent Collective Action Clauses or Exit Consent clauses can solve the coordination problem among the creditors. We thus abstract from the coordination problem among the creditors and focus on the coordination problem between the creditors and the government.

Our work also relates to Bolton and Jeanne (2007), in which the government decides whether to finance borrowing with bank loans or bond loans taking as given the ex-post restructuring outcomes of either financing. In particular, they assume that bond loans are more difficult to restructure than bank loans, which is at odds with the data. Our work takes the financing forms as given, and studies their renegotiation outcomes. We find that bond loans are associated with shorter negotiation and higher total welfare.

The paper is organized as follows. Section 2 presents briefly some empirical background for sovereign debt renegotiation. Section 3 studies the bank loan renegotiation, and Section 4 studies the bond renegotiation. Section 5 examines the impact of the renegotiation outcomes on ex-ante borrowing and investment decisions. We conclude in Section 6.

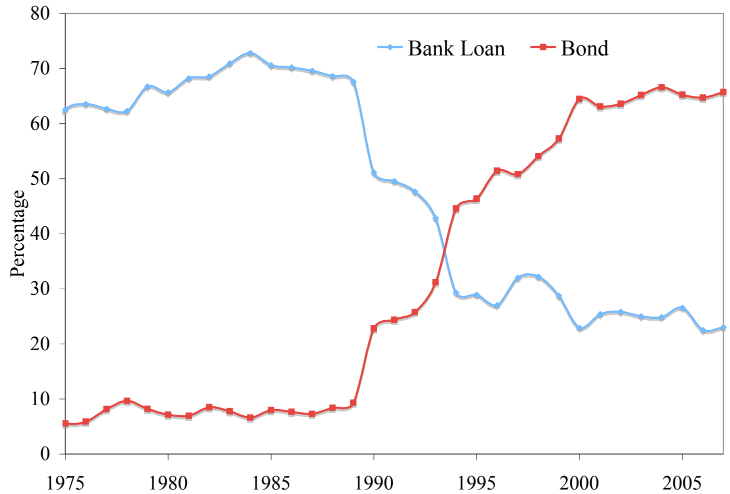
2 Empirical Background

The sovereign debt structure of developing countries has evolved over time from illiquid bank loans to liquid bonds. Figure 1 plots the share of bank loans and bonds in the public and publicly guaranteed debt from private creditors for the developing countries. Before 1990, these countries borrow mainly from commercial banks in advanced economies in terms of syndicated bank loans. The bank loans are customized and rarely traded. The scale of sovereign bonds is minimal. Under the Brady plan (an effort to resolve the 1980s debt crisis), the exchange of commercial bank loans for tradable bonds promoted the development of the secondary market for developing country bonds in the early 1990s. This development sparked a burst of bond issuance by the developing countries. In the 1990s, bonds become the dominant form of private lending to the developing countries.

Sovereign debt renegotiation is usually lengthy. More importantly, the change in the sovereign debt structure is accompanied with a reduction in the renegotiation length. We categorize sovereign defaults on private creditors since 1975 into two groups: bank debt and bond debt. Table 5 reports the summary

[1] Delays arise because both the debtor and the creditors prefer to settle under a good future shock to split a bigger "pie".

Figure 1: Bank Loans and Bonds



Data Source: World Bank's Global Development Finance Database.

statistics on the renegotiation length for each group.[2] The renegotiation process for bank loans is very lengthy, taking on average about 9 years. In contrast, the renegotiation process for bonds is much faster, taking on average about 1 year.

Table 1: Duration of Sovereign Debt Renegotiation, Years

	Mean	Median	Std	Maximum	Minimum	Episodes
All default episodes since 1975						
Bank debt	9.09	7.90	6.00	24.00	0.70	68
Bond debt	1.21	1.10	1.33	4.00	0.00	15
All default episodes since 1990						
Bank debt	4.65	4.70	3.04	12.00	0.70	23
Bond debt	1.21	1.10	1.33	4.00	0.00	15

Data Source: Benjamin and Wright (2008) and Standard & Poor's.

Furthermore, the renegotiation length is still shorter for bonds than for bank loans when we examine two subsamples. We first restrict the sample to the defaults that occur after 1990 since all defaults on the bond debt occur after 1990. As shown in Table 5, the average renegotiation length of bond loans becomes smaller, 4.65 years, but is still longer than that of bonds, 1.29 years. We then examine the sample of countries which have defaulted on both bank debt and bond debt. The results are reported in Table 2. For all these countries, the bond renegotiation is shorter than the bank loan renegotiation.

To understand the duration of sovereign debt renegotiation, we describe briefly empirical renegotiation

[2]For details see Data Appendix.

Table 2: Duration of Debt Renegotiation for Selected Countries, Years

Country	Length		Country	Length	
	Bank Debt	Bond Debt		Bank Debt	Bond Debt
Argentina	11.2	3.6	Paraguay	7.6	1.4
Ecuador	12.3	1.4	Russia	6.0	2.3
Ivory Coast	15.2	4.0	Uruguay	1.1	0.0
Nigeria	10.4	0.0	Venezuela	1.0	0.7

processes.[3] For bank loans, the negotiation starts with the debtor country to mandate a leading bank to organize a *Bank Advisory Committee* (BAC), which usually has representatives from major banks on board.[4] The debtor country then submits a proposal for the restructuring terms it would like to obtain. The BAC sends an economic subcommittee to the debtor country. After thoroughly researching on the debt country's repayment capacity, the economic subcommittee submits a report, based on which the BAC decides whether to accept or reject the debtor country's offer. If the offer is rejected, the debtor country needs to revise the proposal. This process repeats until both parties reach an agreement. Next, the BAC sends the agreement to all participating banks. If a critical mass of creditors respond positively (more than 95% of the outstanding debt), the deal is finalized. This workout process for resolving debt default is also labeled as the *London Club*.

The renegotiation process for bonds is similar. The bond holders organize a committee, which conducts research on the debtor and facilitates the negotiation. The debtor government proposes a restructuring plan and all the bond holders vote on it. If a critical mass of the bond holders approve, the proposal is passed and finalized. Otherwise, the government has to revise the proposal until it passes. Bonds issued under the english law include the *Collective Action clauses* allowing supmajority of the bond holders to change the terms of payment and to make the new terms binding on the minority. Bonds issued under the New York law typically require unanimous consent to make similar changes. In practice, however, these bonds often include the *Exit Consent clauses* allowing majority of creditors to change non-financial terms of the old bonds. For example, dropping the waiver of sovereign immunity, the new bond holders make the old bonds less liquid. Thus, the relevant critical mass in practice is usually the majority rule.

The above renegotiation processes demonstrate that two potential coordination problems might lead to delays in reaching agreements. One is the coordination problem between the debtor country and the creditors. The other is the coordination problem among the creditors. Empirically, the second coordination problem is less severe. In the renegotiation of bank loans, major creditors are already in the BAC, and the small holdouts can be resolved by either servicing them on the original schedule or buying them back. In the renegotiation of bonds, the Collective Action clauses or the Exit Consent clauses in the bond contracts

[3]For an excellent description of sovereign debt renegotiation, see Rieffel (2003).

[4]For efficiency consideration, the number of members is usually kept small, seldom higher than 15.

resolves most of individual holdup problems.[5] Thus, we focus on the coordination problem between the debtor country and the creditors in this work.

The information friction is one of the most important causes for the coordination problem between the debtor country and the creditors. The debtor country might have private information about its repayment capacity. The creditors might have private information about their willingness and tolerance of a debt reduction. The information problem at the debtor country side is solved to a large extent by the comprehensive analysis provided by the IMF and the BAC or the bond holders' association. The private information on the creditors' side, however, might be severe enough to cause the lengthy renegotiation. Thus, we focus on the private information on the creditors' side.

3 Renegotiation of Bank Loans

Syndicated bank loans from commercial banks are the dominant form of private lending to developing countries before 1990. These bank loans are rarely traded and difficult to restructure: the renegotiation takes on average about 9 years. We model the renegotiation of bank loans with a dynamic non-cooperative bargaining game with one-sided incomplete information as Fudenberg et al. (1985). To highlight mechanisms affecting the renegotiation outcomes, we focus on the ex-post renegotiation in Section 3 and 4, and study the ex-ante sovereign borrowing and default decisions in Section 5.

3.1 The Model

There are two parties in the model: the bank (the creditor) and the government (the debtor). At date 0, the government defaults on its bank debt and starts to negotiate with the bank. Assume that the government has a deterministic output process: $y_t = y$ for any t. In each period, the government proposes a restructuring plan that specifies a per period payment b to the bank. The bank either accepts or rejects the proposal. If the proposal is accepted, the renegotiation ends: the government has a per period payoff $y - b$, and the creditor has a per period payoff b. Otherwise, the government loses its output by a fraction γ,[6] and the bank can capture a fraction s of the output loss.[7] The renegotiation continues to the next period until the two parties reach an agreement.

Both parties have a discount factor $\beta < 1$ and maximize the expected payoff. The government obtains per-period payoff $(1-\gamma)y$ regardless of whether the proposal is accepted, and negotiates with the bank to split per-period payoff γy. Clearly, the government never makes an offer larger than γy. Moreover, the

[5] For the bond structuring of Pakistan, Ukraine, Ecuador, Russia and Uruguay, over 90% of the creditors accept the debtor's initial offer, and the deal goes through in less than 1 year. The theoretical work of Haldane et al. (2005) shows that the Collective Action clauses solve the coordination problem among the creditors.

[6] This loss in output could come from various channels: lose access to financial markets, lose trade credits, or disruption of the domestic financial systems.

[7] Following Bulow and Rogoff (1989), we assume that the bank seizes some payoff during the renegotiation to capture the idea that firms in the debtor country have to pay the bank higher fees to obtain trade credits or conduct transactions while the government is in arrears on its bank debt.

bank never accepts an offer lower than $s\gamma y$, and we interpret s as the reservation value of the bank in the renegotiation. Without loss of generality, we subtract the government per-period payoff by $(1-\gamma)y$.

We assume that the bank has private information about s, and the government observes only its distribution: s is uniformly distributed on $[s_l, s_h] \in [0, 1)$. The information asymmetry can be understood as follows. The bank obtains sufficient information about the government before making the loans and while monitoring the loans. The government, however, has little information about the reservation value of the bank.

In each period t, the information set of the government is a history of rejected offers $h_t = \{b_1, b_2, ..., b_{t-1}\}$, and the information set of the bank is the same history concatenated with the current offer (h_t, b_t). A system of beliefs for the government is a mapping from his information set into a probability distribution g_t over s (let G_t denote the cumulative distribution). The government's strategy maps his belief g_t into an offer b_t. The bank's strategy maps his information set into either rejection or acceptance. We define a *perfect Bayesian equilibrium* as follows.

Definition 1. A *perfect Bayesian equilibrium* is a system of beliefs for the government, and a pair of strategies for the government and the bank, such that the government's beliefs are consistent with Bayes' rule (whenever is is applicable) and the strategies of the government and the bank are optimal after any history given the current beliefs.

To characterize any equilibrium, we need to derive the government's belief g_{t+1} about the bank's reservation under any history of the rejected offers $h_t = \{b_j\}_{j=1}^t$. Given the government strategy, the bank rejects offer b_j if and only if his reservation value above is $\underline{s}_j$. This is because a bank with higher reservation derives a higher payoff from rejecting the offer, but derives the same payoff from accepting the offer. Therefore, the government's posterior belief at date $t+1$ is a uniform distribution on interval $[\underline{s}, s_h]$, where $\underline{s}$ is the highest cutoff reservation among $\{\underline{s}_j\}_{j=1}^t$. That is, the belief is truncated from below each period if the offer is rejected. Thus, we characterize the government posterior belief with one number $\underline{s}$.

Dynamic bargaining games with one-sided incomplete information typically have a plethora of equilibria mainly because a perfect Bayesian equilibrium imposes no restrictions on the uninformed party's beliefs following out-of-equilibrium moves.[8] In our bargaining game, however, there exists a unique perfect Bayesian equilibrium. The key assumption is $s_h < 1$, which implies that the government obtains a sure payoff $(1-s_h)\gamma y$ by ending the renegotiation right way with offer $s_h\gamma y$. The potential surplus that the government might hope to extract $(1-\underline{s})\gamma y$ eventually becomes insignificant compared to the sure payoff. Thus, the game always ends in finite periods, which guarantees the existence and the uniqueness of the equilibrium. We summarize these results in Proposition 1.

[8]See Ausubel et al. (2002) for a detailed discussion.

Proposition 1. *If $s_h < 1$, there exists a unique perfect Bayesian equilibrium and the renegotiation ends in finite period T.*

Proof: See Fudenberg et al. (1985).

The government has incentives to reach an agreement with the bank as soon as possible to avoid the output drop and discounting of payoffs. If there is no private information, the agreement is reached in the first period in this deterministic environment: the government offers $b = s\gamma y$ per period and captures the entire surplus from the renegotiation. Thus, a crucial feature of the renegotiation is missed: equilibrium delays in agreements. With private information, the model generates equilibrium delays because the government does not want to make a proposal too high in early stages of the renegotiation to miss the likelihood that the bank has a small reservation value.

In the bargaining game, we assume that the government makes a take-it-or-leave-it offer to the bank each period. Allowing the bank to make an offer will greatly complicate the bargaining game, because the signaling mechanism adds to the complexity when the informed party makes the offers. The game will in general have multiple (or a continuum of) equilibria. Here we deliver the essences of bargaining without invoking this complication.

3.2 Characterization of Equilibrium

The unique equilibrium is characterized inductively. We start by solving the last period game, in which we impose that the government must offer $s_h\gamma y$ to end the renegotiation. In fact, whenever the government's belief about the bank's reservation value becomes high enough, the government is willing to offer $s_h\gamma y$. We then work backward on the number of periods remaining and the government's belief simultaneously until the government's belief is equal to or below s_l. By doing so, we can construct the strategies of both the government and the bank. The equilibrium outcome can be derived accordingly.

We now provide some details of this characterization. Let's start with the last period $t = T$. The government proposes $B_T^*(\underline{s}) = s_h\gamma y$ for any $\underline{s}$ to end the renegotiation right way. The per-period payoff of the government is $V_T^*(\underline{s}) = (1 - s_h)\gamma y$, and the per-period payoff of the bank with reservation s is $s_h\gamma y$.

We next proceed backward until the first period. In any period $t < T$, consider an offer b_t. The per-period payoff of the bank with reservation s is b_t if she accepts today, and $(1-\beta)s\gamma y + \beta B_{t+1}^*(\underline{S}_{t+1}(b_t))$ if she accepts tomorrow, where $\underline{S}_{t+1}(b_t)$ gives the government's update on the lowest reservation value if the offer b_t is rejected. The bank decides whether to accept or to reject to maximize his payoff. In particular, the bank with reservation $\underline{S}_{t+1}(b_t)$ is indifferent between accepting today or tomorrow, that is, $\underline{S}_{t+1}(b_t)$ solves

$$b_t = (1-\beta)\underline{S}_{t+1}(b_t)\gamma y + \beta B_{t+1}^*\left(\underline{S}_{t+1}(b_t)\right). \tag{1}$$

The government then chooses b_t to maximize the per-period welfare

$$V_t(\underline{s}) = \max_{b_t} \Lambda_t(\underline{s}, b_t)(\gamma y - b_t) + (1 - \Lambda_t(\underline{s}, b_t))\beta V^*_{t+1}(\underline{S}_{t+1}(b_t)), \tag{2}$$

where $\Lambda_t(\underline{s}, b_t)$ denotes the acceptance probability of offer b_t, given by $(\underline{S}_{t+1}(b_t) - \underline{s})/(s_h - \underline{s})$ under the uniform distribution. A higher offer will increase the probability of the acceptance, but lower the acceptance payoff of the government. We denote the optimal offer by $B_t(\underline{s})$.

We compute the cutoff belief $\hat{\underline{s}}_{t+1}$, under which the government is indifferent between ending the renegotiation in period t or after period t, by solving $V^*_{t+1}(\hat{\underline{s}}_{t+1}) = V_t(\hat{\underline{s}}_{t+1})$. Thus, for any $\underline{s} \geq \hat{\underline{s}}_{t+1}$, the government prefers to end the renegotiation after period t, and so we update $B^*_t(\underline{s}) = B^*_{t+1}(\underline{s})$. For any $\underline{s} < \hat{\underline{s}}_{t+1}$, the government prefers to end the renegotiation in period t with proposals following $B_t(\underline{s})$. On the other hand, the government has to offer at least $\hat{b}_t$, given by $\underline{S}_{t+1}(\hat{b}_t) = \hat{\underline{s}}_{t+1}$, to ensure ending the renegotiation within $T - t$ periods. Thus, we update the government's optimal proposal $B_t(\underline{s})$ to $B^*_t(\underline{s})$ as follows:

$$B^*_t(\underline{s}) = \begin{cases} B^*_{t+1} & \text{if } \underline{s} \geq \hat{\underline{s}}_{t+1} \\ B_t(\underline{s}) & \text{if } \underline{s}_t < s < \hat{\underline{s}}_{t+1} \\ \hat{b}_t & \text{if } s \leq \underline{s}_t \end{cases} \tag{3}$$

where $\underline{s}_t$ is given by $B_t(\underline{s}_t) = \hat{b}_t$. We also update the government's welfare accordingly and denote it by $V^*_t(\underline{s})$. We proceed the above process until we have $\hat{\underline{s}}_1 \leq s_l$.

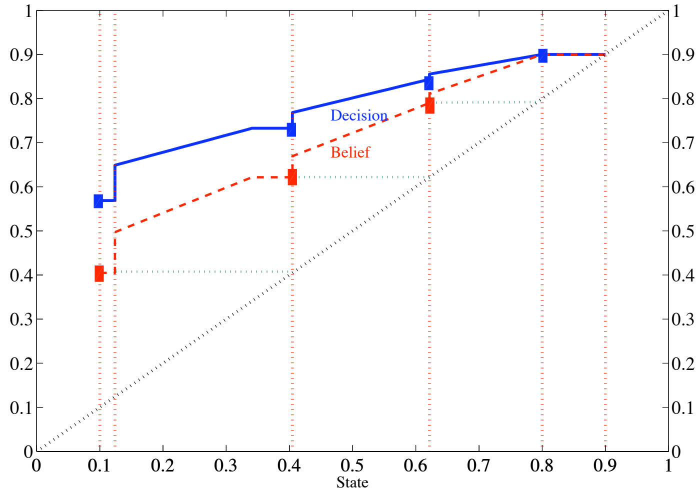
Several characteristics of the optimal strategies deserve attentions. First, the government's proposal (weakly) increases with the belief $\underline{s}$. Second, the government's posterior belief increases with the rejected offer. Third, if the bank has lower reservation, he accepts earlier in the renegotiation.

After deriving the optimal strategies of both the government and the bank, we generate the unique Perfect Bayesian Equilibrium outcomes. The government proposes a sequence of offers given by $\{b^*_1(s_l)$, $b^*_2(\underline{S}_2(b^*_1))$, ..., $b^*_{T-1}(\underline{S}_{T-1}(b^*_{T-2}))$, $b^*_T(\underline{S}_T(b^*_{T-1}))\}$ and $b^*_T(\underline{S}_T(b^*_{T-1})) = s_h$. The bank accepts at date t when his reservation value falls between $\hat{\underline{s}}_t$ and $\hat{\underline{s}}_{t+1}$. Clearly, the higher the reservation value, the longer is the renegotiation and the higher is the repayment offer.

To illustrate the equilibrium strategies and outcomes transparently, we present a numerical example, where $\beta = 0.5$, $[s_l, s_h] = [0.1, 0.9]$, and γy is normalized to one. We plot the optimal strategy of the government and his belief updates in Figure 2. The solid line presents the optimal proposals of the government as a function of his belief $\underline{s}$. The dashed line shows the government's new belief about $\underline{s}$ if his offer $b_t(\underline{s})$ is rejected. The vertical, dashed lines indicate the period cutoffs. The maximum renegotiation length in this case is four periods.

We trace out the equilibrium proposals with blue squares and the equilibrium belief cutoffs with red squares in the figure. In period 1, the government proposes $b_1(s_l) = 0.56$. If the bank's reservation is below $\underline{S}_2(b_1) = 0.4$, the proposal is accepted and the renegotiation ends. Otherwise, the bank rejects the offer, and the government updates his belief to $\underline{S}_2(b_1)$. In period 2, the government proposes $b_2(\underline{S}_2(b_1)) = 0.72$,

Figure 2: Government's Optimal Strategy



which is accepted by the bank with reservation below $\underline{S}_3(b_2) = 0.6$. In period 3, the government proposes $b_3(\underline{S}_3(b_2)) = 0.81$, which is accepted by the bank with reservation below $\underline{S}_4(b_3) = 0.76$. In period 4, the government proposes $b_4(\underline{S}_4(b_3)) = 0.9$, and the renegotiation ends.

3.3 Duration of Bank Loan Renegotiation

Now we study our key interest: duration of the bank loan renegotiation. In particular, we are interested in how the degree of the information friction impacts the renegotiation length. We find analytically that the maximum renegotiation length increases with the degree of the information friction. Also, we find numerically that the expected renegotiation length displays an increasing trend as the information friction rises.

We measure the degree of the information friction, denoted by Ψ, as follows:

$$\Psi = \frac{1 - s_l}{1 - s_h}, \tag{4}$$

and a higher Ψ indicates a higher degree of the information friction. In the proposition below, we demonstrate that the maximum renegotiation length increases with the degree of the information friction Ψ. The economic intuition for this result is the following. $1 - s_l$ is the largest possible payoff of the government from the renegotiation, and $1 - s_h$ is the sure payoff if the government ends the renegotiation right way. A larger Ψ means that the maximum potential payoff increases relative to the sure payoff. Thus, the government has more incentives to use the costly equilibrium delays to screen the bank's reservation.

Proposition 2. *The maximum renegotiation length increases with the degree of the information friction Ψ.*

Proof: See Technical Appendix.

Following Proposition 2, we can immediately show that the maximum renegotiation length increases as s_h decreases or s_l increases. Moreover, we can also easily show that the maximum renegotiation length increases if the interval of the potential reservation values shifts to the right.[9] This is because the degree of the information friction Ψ increases as the interval $[s_l, s_h]$ shifts to the right to $[s_l + h, s_h + h]$ with $h > 0$.

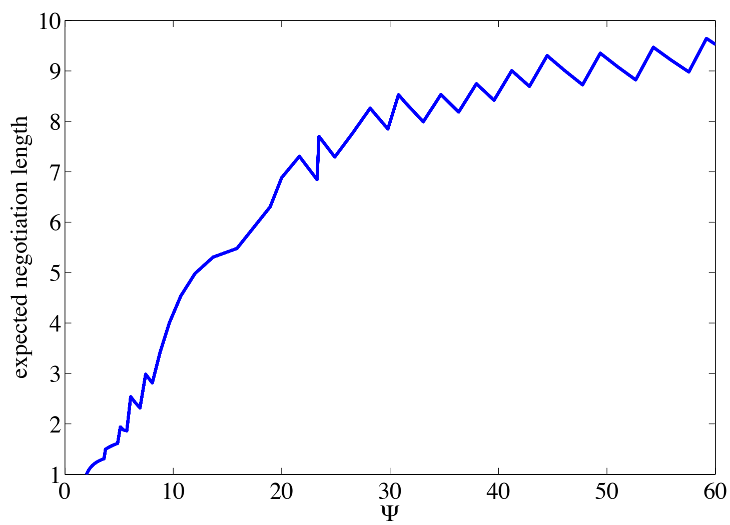
We next look at the expected renegotiation length. For each initial belief $[s_l, s_h]$, we have a sequence of the government's offers $\{b_1, b_2, ..., b_T\}$ and a sequence of the belief updates $\{s_1, s_2, ..., s_T, s_{T+1}\}$, where $s_1 = s_l$, $s_{T+1} = s_h$, and $s_{t+1} = \underline{S}_{t+1}(b_t)$ for any $t = 1, ...T - 1$. We define the expected renegotiation length T^e as

$$T^e = \sum_{t=1}^{T} \mathcal{P}_t t, \tag{5}$$

where $\mathcal{P}_t$ denotes the probability of the renegotiation ending in period t, and is given by $(s_{t+1} - s_t) / (s_h - s_l)$.

Due to complexity of the solution, we characterize the expected renegotiation length numerically. We set $\beta = 0.98$ and $\gamma y = 1$, and explore how the expected renegotiation length varies with the information friction Ψ. The results are illustrated in Figure 3. As Ψ increases, the expected renegotiation length displays an upward trend. That is, as the information friction becomes more severe, the renegotiation tends to take longer to settle. The wiggles are driven by discrete time periods. When an increase in Ψ does not change the maximum renegotiation length, $\mathcal{P}_1$ increases, while $\mathcal{P}_t$ decreases for any other t. Thus, the expected renegotiation length decreases. When an increase in Ψ drives up the maximum renegotiation length, the expected renegotiation length increases.

Figure 3: Expected Renegotiation Length



[9] This result is important when we discuss the renegotiation outcomes for bonds in the next section.

In sum, we use a classic dynamic bargaining game with incomplete information to model the renegotiation of bank loans. The information friction generates equilibrium delays in reaching agreements in the renegotiation. Moreover, we find that the delays become shorter as the information friction decreases. In the next section, we model the renegotiation of bonds with the secondary market. The secondary market trading serves an important role in reducing the information frictions, and thus the delays are greatly shortened.

4 Renegotiation of Bonds

After 1990, bond issuances become the dominant form of sovereign borrowing in developing countries. Sovereign bonds are standardized, issued to the public and traded on the secondary market. The renegotiation process of bonds is short, taking on average about one year, compared to 9 years for structuring bank loans. We argue that the secondary market plays an important information-revelation role in reducing the renegotiation length. In this section, we model the renegotiation between the government and a continuum of bond holders with the presence of the secondary market. We design the model as close as possible to the one of bank loans, while incorporating the secondary market. This way we can highlight the role of the secondary market on the renegotiation outcomes for bonds.

4.1 The Model with the Secondary Market

There are two types of agents in the economy: a sovereign government and a continuum of creditors of measure one. For simplicity, we assume all the creditors hold one unit of bonds. After the government announces default at date 0, the secondary market opens and the creditors trade at the market price p. In the trading stage, the government and the creditors know that creditors' reservation s is drawn from a uniform distribution $[s_l, s_h]$. Each creditor receives a signal z about s, where $z = s + \sigma_z \varepsilon$ with ε uniformly distributed on $[-1, 1]$. In date 1, the creditors with bonds observe their common reservation value s and negotiate with the government. The renegotiation follows the same process as that for bank loans. All the agents have the same discount factor β and maximize the expected payoff.

Each creditor can hold on to the bond or trade on the secondary market: either selling or buying one unit of bonds.[10] The payoff of selling is the market price p. The payoff of holding or buying depends on the expected payoff from the renegotiation. Conditional on his private signal z and the public information p, the creditor calculates the expected payoff and makes the trading decision.

A random fraction α of the creditors are noisy creditors; they sell their bonds regardless of their signals. The ratio of noisy and non-noisy creditors $\alpha/(1-\alpha)$ is given by

$$\frac{\alpha}{1-\alpha} \equiv \sigma_\eta \eta, \tag{6}$$

[10]The assumption on the upper bound of trading makes our analytics simple and transparent, and it is not essential for our main findings.

where η is a random variable and uniformly distributed on $[0, 1]$, and $0 < \sigma_\eta < 1$. A higher σ_η implies a larger measure of noisy creditors. Nonetheless, the measure of the noisy creditors is less than that of the non-noisy creditors.

At period 1, the reservation value s is revealed to all the creditors with bonds, but not to the government. The government starts the renegotiation and proposes a per-period repayment plan b_t in each period t. Every creditor either accepts or rejects the offer. If a critical fraction κ of the creditors accept, the bargaining process ends: the government has a per period payoff $y - b$, and each creditor has a per-period payoff b. If a less-than-κ fraction of creditors accept, the agreement is not reached and the bargaining continues to the next period: the government loses a γ fraction of its output, and the creditors capture a fraction s of the output loss.

In the model, we could vary κ to capture any critical mass. Our model results, however, are independent of κ. This is because all the creditors are ex-post identical with the same reservation s and they either all accept or all reject. We abstract from the coordination problem between creditors, such as free-riding in renegotiation costs and strategic hold-outs due to two reasons. One is that we want to focus on the coordination problem between the debtor country and the creditors. The other is that the coordination problem is less relevant with the Collective Action clauses or the Exit Consent clauses as shown in Shin et al 200X.

We restrict the trading strategy of the creditors to be *monotonic*: the creditor buys whenever his signal z is more than $\hat{z}(p)$ and vice versa. We define the *monotonic perfect Bayesian equilibrium* below.

Definition 2. A *monotonic perfect Bayesian equilibrium* consists of a market price p, beliefs of the government as a function of p, beliefs of the creditors as a function of p and signal z, and a monotonic trading strategy of the creditors $\hat{z}(p)$ in the trading stage, and a pair of strategies for the government and the creditors and a system of beliefs for the government in the renegotiation stage, such that

- In the renegotiation stage, the government's beliefs are consistent with Bayes' rule (whenever is applicable) and the strategies of the government and the creditors are optimal after any history given the current beliefs.
- In the trading stage, the monotonic trading strategy is optimal for all creditors given their beliefs, updated using the market price p and private signals z. The government also updates his belief using the market price p.
- The secondary markets clear at price p.

4.2 Equilibrium with the Secondary Market

Trading on the secondary market influences sovereign debt renegotiation outcomes. Without the secondary market, the renegotiation outcomes of bond restructuring are the same as those of bank loan restructuring. With the secondary market, the government updates his belief about the reservation value of the creditors using the observed secondary market price. The price is informative because it aggregates the signals according to which the creditors trade. We now describe the information revelation mechanism of the secondary market.

According to the creditors' monotonic trading strategy $\hat{z}(p)$, the excess demand of non-noisy creditors $X(s,p)$ is given by

$$X(s,p) = (1-\alpha)\left[P\left(z > \hat{z}\left(p\right)\middle| s\right) - P\left(z \leq \hat{z}(p)|s\right)\right], \tag{7}$$

where $P\left(z > \hat{z}(p)\middle| s\right)$ denotes the probability of signals above $\hat{z}(p)$, i.e., the amount of bonds demanded, and $P\left(z \leq \hat{z}(p)|s\right)$ the amount of bonds supplied. Since z is uniformly distributed on $[s-\sigma_z, s+\sigma_z]$, simple algebra delivers

$$X(s,p) = (1-\alpha)\frac{s-\hat{z}(p)}{\sigma_z}. \tag{8}$$

The bond supply from the noisy creditors is the fraction of the noisy creditors $0 < \alpha < 1$. From the market clearing condition, in equilibrium we have

$$\frac{s-\hat{z}(p)}{\sigma_z} = \frac{\alpha}{1-\alpha} = \eta\sigma_\eta, \tag{9}$$

or equivalently,

$$s = \hat{z}(p) + \sigma_z\sigma_\eta\eta. \tag{10}$$

Therefore, the government infers that s is uniformly distributed on $[\hat{z}(p), \hat{z}(p)+\sigma_z\sigma_\eta]$, when observing the market price p. Together with his prior, the government updates his belief about s to the interval $[s_l^G, s_h^G]$, where $s_l^G = \max\{\hat{z}(p), s_l\}$ and $s_h^G = \min\{\hat{z}(p)+\sigma_z\sigma_\eta, s_h\}$. The government then forms his renegotiation strategy accordingly, as discussed in the previous section.

The creditors also use the market price to form their expected payoff from the renegotiation and make the trading decisions. When observing p, the creditors know that in the renegotiation stage the government will propose according to his new prior updated with $\hat{z}(p)$. The creditors compute the payoff $W^N(s, \hat{z}(p))$ for each reservation value s. His belief about s is updated using his signal z on the government's belief based on the public signal. The expected renegotiation payoff is thus given by $E_s\left[W^N(s,\hat{z}(p))|\hat{z}(p), z\right]$.[11] The payoff to sell is p, the payoff to hold is $E_s\left[W^N(s,\hat{z}(p))|\hat{z}(p), z\right]$, and the payoff to buy is $-p+2E_s\left[W^N(s,\hat{z}(p))|\hat{z}(p), z\right]$. Clearly, holding is always weakly dominated by either selling or buying.

[11] We assume that the trading stage is so short that there is no discounting for the renegotiation payoff. The only purpose of this assumption is for conveniency when we compare welfare across the two games.

In the monotone trading strategy, the creditor with the cutoff signal must be indifferent between buying and selling. Thus, the secondary market price p has to equal to the expected payoff of the critical creditor, that is,

$$p = E_s \left[W^N(s, \hat{z}(p)) | \hat{z}(p), \hat{z}(p) \right]. \tag{11}$$

For the monotone trading strategy to be optimal, we also need that the creditors with higher signals (weakly) prefer buying to selling, and vice versa. Specifically, we need that any creditor with signal $z > \hat{z}(p)$ prefers to buy if and only if

$$p \leq \beta E_s \left[W^N(s, \hat{z}(p)) | \hat{z}(p), z \right]. \tag{12}$$

This condition holds because $W^N(s, \hat{z}(p))$ is (weakly) increasing in s and a higher signal z implies that s is likely to be higher.

For each realization of (s, η), there is a unique pair of $(\hat{z}, p)$, which characterizes the monotonic perfect Bayesian equilibrium. Specifically, there is a unique cutoff signal $\hat{z}$ that clears the market by solving equation (10), and the market price equals to the expected payoff of the critical creditor by solving equation (11). Given the cutoff signal $\hat{z}$, the government's and the creditors' renegotiation strategies is uniquely pinned down. This implies that there is a unique value of the expected payoff of the critical creditor, and so a unique market price p. Thus, the monotonic perfect Bayesian equilibrium exists and is unique. We summarize this result in the following proposition.

Proposition 3: *There exists a unique monotonic perfect Bayesian equilibrium.*

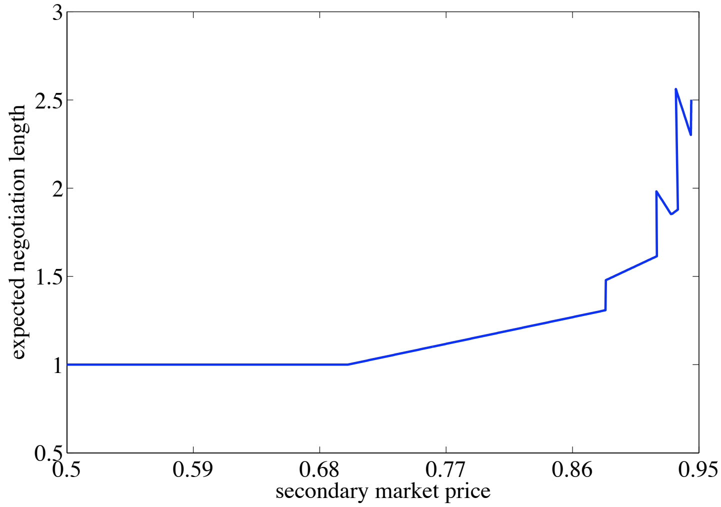
Proof: See Technical Appendix.

The secondary market price plays an important role in determining the equilibrium delays. The government uses the price to update his belief about the creditors' reservation and form the renegotiation strategies. We illustrate the relation between the secondary market price and the expected renegotiation length numerically in Figure 4 with the benchmark parameters set as $\gamma y = 1$, $[s_l, s_h] = [0, 0.99]$, $\beta = 0.98$, $\sigma_z = 0.5$, and $\sigma_\eta = 0.5$. We find that overall a higher price is associated with longer renegotiation. An increase in p tends to increase the cutoff signal $\hat{z}$, shifting the set of potential reservation values to the right. This implies an increase in the degree of the information friction Ψ. As a result, the maximum renegotiation length increases with the secondary market price, according to Proposition 2. The expected renegotiation length displays a similar pattern.

4.3 Information Revelation of Secondary Market

To highlight the role of the secondary market, we compare the duration of the bond renegotiation with that of the bank loan renegotiation. We first show analytically that the secondary market reduces the equilibrium

Figure 4: Market Price and Expected Renegotiation Length



delays in reaching agreements. In particular, when there is no noisy trader, the secondary market price is perfectly revealing and there is no equilibrium delay. We then illustrate numerically the quantitative impacts of the secondary market on the renegotiation length.

The maximum duration of the bond renegotiation is (at least weakly) shorter than that of the bank loan renegotiation. The key to this result is that as long as the secondary market price is somewhat informative, the government will form an updated belief $[s_l^G, s_h^G]$ about the creditor's reservation value s. The new belief $[s_l^G, s_h^G]$ is a subset of the ex-ante belief $[s_l, s_h]$. Thus, the information friction Ψ decreases with the secondary market. As a result, the maximum renegotiation length under the new belief is shortened, as shown in Proposition 2. We summarize this finding in the following proposition.

Proposition 4: *The maximum renegotiation length of the bond renegotiation is always shorter than or equal to that of the bank loan renegotiation.*

Proof: See Technical Appendix.

Now consider the extreme case that the secondary market is perfect with no noise. The market price p then perfectly reveals the reservation value s. The government proposes $b_1 = s\gamma y$ and the creditors accept at date 1. This is because the government makes all the proposals and the creditors have no bargaining power. In this case, the renegotiation outcomes are efficient: no equilibrium delays in reaching agreements and no loss in the total surplus. These findings are summarized in Proposition 5 below.

Proposition 5: *If $\sigma_\eta = 0$, the government updates his belief s to be $\hat{z}(p)$ and proposes $s\gamma y$ each period, and the creditors accept the offer in period 1.*

Proof: See Technical Appendix.

We next compare the expected duration of the bond renegotiation with that of the bank loan renegotiation quantitatively. Consider the benchmark parameter values: $\gamma y = 1$, $[s_l, s_h] = [0, 0.99]$, $\beta = 0.98$, $\sigma_z = 0.5$, and $\sigma_\eta = 0.5$. The expected renegotiation length for bank loans is 9.1; the secondary market trading reduces it to 1.18. The duration of the bond renegotiation depends on the noises on the secondary market: σ_z and σ_η. We vary first σ_z and then σ_η while keeping all the other parameters fixed, and report the expected renegotiation length under column T^e in Table 3. We note that the expected length of the bond renegotiation is always shorter than that of the bank loan renegotiation. Moreover, as the information becomes more precise, i.e. as σ_z or σ_η decreases, the duration of the bond renegotiation becomes shorter.

Table 3: Comparative Analysis over Signal and Noise

σ_z	T^e	$w^c + w^g$	w^c	w^g	σ_η	T^e	$w^c + w^g$	w^c	w^g
0.10	1.03	1.00	0.52	0.48	0.10	1.03	1.00	0.52	0.48
0.50	1.83	1.00	0.61	0.39	0.50	1.83	1.00	0.61	0.39
0.90	2.92	1.00	0.68	0.32	0.70	2.36	1.00	0.64	0.35
3.00	7.04	0.98	0.81	0.16	0.90	2.92	1.00	0.68	0.32

4.4 Welfare Analysis

We now study the welfare implications of the bank loan renegotiation and the bond renegotiation. In particular, we are interested in how the secondary market affects the total welfare and the allocation of the welfare across the government and the creditors. The welfare analysis has important policy relevance. It helps us evaluate which form of sovereign borrowing increases the total surplus, and which form is friendly to the debtor country or to the creditors. Answers to these questions are fundamental for reforming the sovereign debt restructuring framework.

In both models of sovereign debt renegotiation, the government and the creditors bargain over the ex-ante surplus $\gamma y/(1-\beta)$. The expected surplus of the government from the renegotiation, denoted by W^g, is given by

$$W^g = \sum_{t=1}^{T} \frac{\beta^{t-1}\mathcal{P}_t(\gamma y - b_t)}{1-\beta}, \tag{13}$$

where $\mathcal{P}_t$ is the probability of the renegotiation ending in period t. The expected surplus of the creditors, denoted by W^c, is given by

$$W^c = \sum_{t=1}^{T} \frac{\beta^{t-1}\mathcal{P}_t b_t}{1-\beta} + \sum_{t=2}^{T} \frac{1-\beta^{t-1}}{1-\beta} \int_{s_t}^{s_{t+1}} s\gamma y dG(s), \tag{14}$$

where the first term is the expected repayment from the government and the second term is the expected payoff during the renegotiation. We denote the share of the government expected surplus in the ex-ante surplus by w^g and the share of the creditors' expected surplus by w^c.

We first study the total expected surplus $w^c + w^g$. When there is no equilibrium delay, there is no loss in the ex-ante surplus, i.e., $w^c + w^g = 1$. Otherwise, some ex-ante surplus is lost during the lengthy renegotiation: $w^c + w^g < 1$. Moreover, the longer is the renegotiation, the more total expected surplus is lost. As a result, the bond renegotiation generates a higher total expected surplus because the duration of the bond renegotiation is shorter than that of the bank loan renegotiation. Under the benchmark parameters, the total expected surplus is 0.96 in the bank loan renegotiation, and 1 in the bond renegotiation. Table 3 reports the total expected surplus for the bond renegotiation under different noise parameters under column $w^c + w^g$. The more precise is the information, the higher the total surplus is.

We next examine the split of the total surplus between the government and the creditors. The bond renegotiation generates a lower creditors' share, but a higher government's share of the total expected surplus, than the bank loan renegotiation. This is because the secondary market reduces the information friction, and leads to a shift of the bargaining power from the creditors to the government. For the benchmark parameters, the creditors' share w^c is 0.88 under bank loans but only 0.61 under bonds, while the government's share w^g is 0.08 under bank loans, but 0.39 under bonds. Table 3 reports w^c and w^g for different noise parameters in the bond renegotiation. We find that a reduction in the information friction leads to a higher w^g and a lower w^c.

Therefore, the bond financing improves the total surplus through reducing the duration of sovereign debt renegotiation. Moreover, the bond financing is more friendly to the debtor country because it lessens the information friction and gives the debtor country more bargaining power. On the contrary, the bank loan financing is more friendly to the creditors by allowing them to a higher share of the total surplus.

5 Ex-Ante Borrowing and Lending

We have analyzed the equilibrium outcomes of the renegotiation of different forms of sovereign borrowing: bank loans versus bonds. Understanding these renegotiation outcomes is important for two key reasons. First, the expected renegotiation outcomes impact directly how the creditors price the debt ex-ante. Second, these renegotiation outcomes also affect the government's borrowing incentives and default decisions. When taking these considerations into account, we evaluate the impact of the form of sovereign borrowing on the total welfare and on the distribution of the welfare between the debtor country and the creditors.

5.1 Model

Consider a sovereign country that borrows to invest in a project in date 0. For an investment level I, the output of the project is $Af(I)$ per period, where A denotes stochastic productivity of the project. In date 1, A realizes with probability π_H to be A_H and with probability π_L to be A_L, and $\pi_L + \pi_H = 1$. The productivity is constant afterward. The function f is assumed to be $f(I) = I^\alpha$. Both the government and the creditors have a discount factor $\beta < 1$.

We first examine the optimal investment with no default risk. The optimal investment, equating the expected return to the risk free rate R, is $I^* = \left(\alpha \bar{A} / \left(R(1-\beta)\right)\right)^{\frac{1}{1-\alpha}}$, where $\bar{A}$ denotes the expected productivity. We refer to I^* as the efficient investment later.

We then examine the optimal investment with default risk on sovereign borrowing. Before the renegotiation stage, both the government and the creditors are informed of only the distribution of the reservation value s. The debt contract is specified by a pair (I, b), where I is the resource that the creditors lend to the government in date 0, and b the amount that the government promises to repay each period from date 1. The government chooses a debt contract to maximize the expected welfare, $\pi_H V_H(I, b) + \pi_L V_L(I, b)$, where $V_j(I, b)$ denotes the welfare under shock $j = H$ or L. After observing A_j, the government decides whether to default by maximizing over the payoffs from repaying and from defaulting:

$$V_j(I, b) = \max\left\{V_j^R(I, b), V_j^D(I, b)\right\}, \quad j = L \text{ or } H,$$

where $V_j^R(I, b)$ denotes the repayment welfare, and $V_j^D(I, b)$ the defaulting welfare.

The defaulting welfare $V_j^D(I, b)$ is the expected welfare of the government from the renegotiation, since the government needs to renegotiate with the creditors. The government receives at least $(1-\gamma)A_j f(I)$ each period after default, and splits $\gamma A_j f(I)/(1-\beta)$ with the creditors. Denote the share that the government expects to receive by w^g, which is independent of the output level.[12] Thus, the expected welfare or the defaulting welfare is

$$V_j^D(I, b) = \frac{(1-\gamma)A_j f(I)}{1-\beta} + \frac{\gamma A_j f(I) w^g}{1-\beta}. \tag{15}$$

The repayment welfare $V_j^R(b, I)$ is simply the lifetime discounted value of output $A_j f(I)$ subtracting the repayment b, and can be written as

$$V_j^R(I, b) = \frac{(1-\gamma)A_j f(I)}{1-\beta} + \frac{\gamma A_j f(I) - b}{1-\beta}. \tag{16}$$

Therefore, the government will default if $b \geq (1-w^g)\gamma A_j f(I)$.[13] Denote the cutoff debt, above which the government chooses to default under shock j, by $\underline{b}_j(I)$. Thus, we have $\underline{b}_H(I) = (1-w^g)\gamma A_H f(I)$ and $\underline{b}_L(I) = (1-w^g)\gamma A_L f(I)$, with $\underline{b}_H(I) > \underline{b}_L(I)$ for any I. The government defaults under both shocks for any contract with $b \geq \underline{b}_H(I)$, defaults under only the low shock for any contract with $b \in [\underline{b}_L(I), \underline{b}_H(I)]$, and defaults under neither shocks for any contract with $b \leq \underline{b}_L(I)$,

Given the government's default decisions, the creditors design the set of debt contracts offered to the debtor country to maximize the profit:

$$\max_{\{I,b\}} \quad \Pi(I, b) = -R(1-\beta)I + b_R$$

[12] We show this result in Lemma A.1. of Technical Appendix.

[13] In this simple ex-ante borrowing game, we abstract from the risk-sharing consideration of default by assuming linear preferences of the government.

subject to

$$b_R = \begin{cases} b & \text{if } b \le b_L(I) \\ \pi_H b + \pi_L \gamma w^c A_L f(I) & \text{if } b_L(I) < b \le b_H(I) \\ \gamma w^c \bar{A} f(I) w^c & \text{if } b > b_H(I) \end{cases} \tag{17}$$

where w^c is the share of $\gamma A_j f(I)/(1-\beta)$ that the creditors expect to obtain from the renegotiation. For $b \in [\underline{b}_L(I), \underline{b}_H(I)]$, the implicit risk premium depends on the renegotiation outcomes, specifically, the creditors' share of the expected payoff w^c. When w^c increases, i.e., when the creditors expect to obtain a larger share in the renegotiation, the risk premium goes down, and vice versa. We assume that the creditors face the Bertrand competition, and the expected profit $\Pi(I, b)$ is driven to zero in equilibrium for any contract (I, b).

Let $\bar{I}_1$ be the highest investment level such that the government will not default under either shock. $\bar{I}_1$ is thus the solution to $R(1-\beta)I = \underline{b}_L(I)$, i.e.,

$$\bar{I}_1 = \left(\frac{\gamma A_L(1-w^g)}{R(1-\beta)}\right)^{\frac{1}{1-\alpha}}.$$

Let $\bar{I}_2$ be the highest investment level such that the government will not default under both shocks, $\bar{I}_2$ satisfies

$$\bar{I}_2 = \left(\frac{\gamma \pi_H A_H(1-w^g) + \gamma \pi_L A_L w^c}{R(1-\beta)}\right)^{\frac{1}{1-\alpha}}.$$

Assume $A_L \le \pi_H A_H$. It is easy to show that $\bar{I}_2 \ge \bar{I}_1$.

We now characterize the equilibrium contracts. The contracts (I, b) offered are given by,

$$b = \begin{cases} R(1-\beta)I & \text{if } I \le \bar{I}_1 \\ (R(1-\beta)I - \pi_L \gamma w^c A_L f(I))/\pi_H & \text{if } \bar{I}_1 < I \le \bar{I}_2 \end{cases}. \tag{18}$$

Note that the creditors never offer a contract that the government will default under both shocks. This is because such contracts generate negative profits. For any contract with $I > \bar{I}_2$, it costs the creditors $R(1-\beta)I$ but only generates $\gamma w^c \bar{A} f(I)$, which is smaller than the cost.

5.2 Optimal Investment and Borrowing

We now analyze the optimal investment and borrowing with default risk. Given the set of the available debt contracts, the government solves the following problem:

$$\max_{b,I} \quad \pi_H V_H(I, b) + \pi_L V_L(I, b)$$

subject to equation (18). We are interested in the case when the government borrows to the level that he will default only under the low shock.[14] In this case, the optimal investment, denoted by I^d, is given by

$$I^d = \begin{cases} \left[\alpha(\bar{A} - \gamma \pi_L A_L(1-w^c-w^g))/\left(R(1-\beta)\right)\right]^{\frac{1}{1-\alpha}} & \text{if } \alpha \le \gamma(1-w^g) \\ \left[\gamma(\pi_H A_H(1-w^g) + \pi_L A_L w^c)/(R(1-\beta))\right]^{\frac{1}{1-\alpha}} & \text{if } \alpha > \gamma(1-w^g) \end{cases}. \tag{19}$$

[14]The condition needed is $\gamma(1-w^g)A_L \le \alpha\bar{A} \le \gamma(1-w^g)\bar{A} - \eta_1$, where $\eta_1 = (1-\alpha)\gamma(1-w^c-w^g)\pi_L A_L \ge 0$.

The optimal investment with default risk I^d depends on the renegotiation outcomes w^c and w^g. Consider the first sub-case where $\alpha \leq \gamma(1 - w^g)$. The optimal investment I^d depends only on the sum of w^c and w^g. When there are no equilibrium delays in reaching agreements, there is no loss in the total surplus, i.e., the sum of w^c and w^g is one. As a result, the optimal investment with default risk I^d is the same as the efficient investment without default risk I^*. When there are equilibrium delays, however, the sum of w^c and w^g is less than one. Consequently, the optimal investment I^d is lower than the efficient investment I^*.

For the sub-case where $\gamma(1-w^g) < \alpha$, the optimal investment I^d depends on the split of the total surplus between the government and the creditors. In particular, I^d increases with the share of the creditors w^c, but decreases with the share of the government w^g. Moreover, the optimal investment is alway lower than the efficient investment I^*.

The quantitative analysis in the previous section shows that the sum of the total renegotiation surplus increases when the country switches the form of financing from bank loans to bonds. At the same time, the share of the government w^g increases, but the share of the creditors w^c decreases. If this comparative analysis falls into the first sub-case, the optimal investment I^d is higher for bond financing than for bank loan financing. Moreover, the bond financing might help the government achieve the efficient investment level I^*. If this comparative analysis falls into the second sub-case, the optimal investment I^d is lower for bond financing than for bank loan financing.

6 Conclusion

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Data Appendix

Debt Composition

World Bank's Global Development Finance database reports the outstanding stock of privately held public or publicly guaranteed debt (PPG) for the developing countries. The PPG debt is classified into three groups by the type of creditors: bonds, commercial bank loans, and other private creditors. All series are in U.S. dollars. The sum of bank loans and bonds accounts for majority of the privately held PPG debt. The share of the PPG debt held by other private creditors is relatively small. In Figure 1, we plot the percentage of bank loans and bonds as the privately held PPG debt over 1975-2007.

Duration of Sovereign Debt Renegotiation

Benjamin and Wright (2008) collects the starting date, the ending date, and the negotiation length for 90 episodes of sovereign debt restructuring. Nonetheless, there is no information about the form of sovereign debt in their report. We supplement the form of sovereign debt for each debt restructuring using Standard & Poor's (2004). Among the 90 episodes reported by Benjamin and Wright, 68 episodes are in the form of bank loans, and 15 episodes are in the form of bonds. We ignore 7 episodes which are domestic debt. We summarize the duration of bank loan and bond renegotiations in the tables below separately.

Table 4: Duration of Sovereign Debt Renegotiation, Bond Loans

Country	Default start	Default end	Length (years)	Country	Default start	Default end	Length (years)
Argentina	2001	2005	3.6	Paraguay	2003	2004	1.4
Ecuador	1999	2000	1.7	Russia	1998	2000	2.3
Ecuador	2000	2001	1.1	Ukraine	1998	2000	1.4
Guatemala	1989	1989	0.0	Uruguay	2003	2003	0.0
Ivory Coast	2000	2004	4.0	Venezuela	1995	1997	2.0
Moldova	1998	1998	0.0	Venezuela	1998	1998	0.0
Moldova	2002	2002	0.5	Venezuela	2005	2005	0.1
Nigeria	2002	2002	0.0				

Table 5: Duration of Sovereign Debt Renegotiation, Bank Loans

Country	Default start	Default end	Length (years)	Country	Default start	Default end	Length (years)
Albania	1991	1995	4.6	Mauritania	1992	1996	4.7
Algeria	1991	1996	5.2	Mexico	1982	1990	7.9
Angola	1985	2004	19.0	Morocco	1986	1990	4.6
Argentina	1980	1990	11.2	Mozambique	1983	1992	10.0
Bolivia	1980	1993	12.4	Myanmar	1997	2003	6.0
Brazil	1983	1994	11.2	Nicaragua	1979	2003	24.0
Bulgaria	1990	1994	4.3	Niger	1983	1991	7.9
Burkina Faso	1983	1996	13.0	Nigeria	1982	1992	10.4
Cameroon	1985	2003	18.0	Pakistan	1998	1999	1.6
Cape Verde	1981	1996	15.7	Panama	1983	1996	12.7
Central African Republic	1983	2004	21.0	Paraguay	1986	1993	7.6
Chile	1983	1990	7.4	Peru	1980	1980	0.9
Colombia	1985	1991	5.3	Peru	1983	1997	14.4
Costa Rica	1983	1990	6.7	Philippines	1983	1992	9.6
Croatia	1992	1996	4.0	Poland	1981	1994	12.9
Dominica	2003	2004	1.0	Romania	1981	1983	1.5
Dominican Republic	1983	1994	10.9	Russia	1991	1997	6.0
Ecuador	1982	1995	12.3	Sao Tome and Principle	1987	1994	7.7
Ethiopia	1991	1999	8.1	Senegal	1990	1990	0.7
Gabon	1986	1994	7.4	Senegal	1992	1996	5.0
Gabon	1999	2004	4.7	Serbia and Montenegro	1992	2004	12.0
Gambia	1986	1990	4.2	Seychelles	2000	2002	2.0
Guinea	1986	1988	2.3	Sierra Leone	1986	1995	9.7
Guinea	1991	1998	8.0	South Africa	1993	1993	0.7
GuineaBissau	1983	1996	13.0	Tanzania	1984	2004	20.3
Guyana	1982	2004	21.5	Togo	1991	1997	7.0
Haiti	1982	1994	12.0	Trinidad and Tobago	1988	1989	2.0
Honduras	1981	2004	23.0	Uganda	1980	1993	13.2
Ivory Coast	1983	1998	15.2	Uruguay	1990	1991	1.1
Jamaica	1987	1993	6.1	Venezuela	1990	1990	1.0
Jordan	1989	1993	4.1	Vietnam	1985	1998	14.0
Kenya	1994	2004	10.0	Yemen	1985	2001	16.5
Macedonia	1992	1997	5.2	Zambia	1983	1994	10.5
Madagascar	1981	2002	20.1	Zimbabwe	2000	2004	4.0

Technical Appendix

Proof of Proposition 2

To prove Proposition 2, we establish two lemmas. Lemma A.1 shows that the belief, the strategy and the welfare of the government have the homogeneity property. Lemma A.2 demonstrates that the cutoff belief is a linear function of s_h. For conveniency of the proofs, we write the welfare and the optimal strategy of the government as functions of $(1-s; 1-s_h)$ instead of $(s; s_h)$, and the belief as a function of $(1-b; 1-s_h)$.

Lemma A.1. *Government's welfare V_t, optimal strategy B_t, and belief function $\underline{S}_{t+1}$ have the homogeneity property. Specifically, for any $\lambda \in (0, 1/(1-s_h))$,*

$$V_t\left(\lambda(1-s); \lambda(1-s_h)\right) = \lambda V_t\left(1-s; 1-s_h\right) \tag{20}$$

$$1 - B_t(\lambda(1-s); \lambda(1-s_h))/\left(\gamma y\right) = \lambda\left[1 - B_t\left(1-s; 1-s_h\right)/\left(\gamma y\right)\right] \tag{21}$$

$$1 - \underline{S}_{t+1}(\lambda(1-b); \lambda(1-s_h)) = \lambda\left[1 - \underline{S}_{t+1}\left(1-b; 1-s_h\right)\right]. \tag{22}$$

Proof: We prove the homogeneity by induction. We first show that the homogeneity holds for the last two periods, $t = T$ and $T-1$. We then prove the homogeneity holds for period n, assuming that it holds for period $n+1$, for any $n \leq T-1$.

For the simplicity of the proofs, we normalize the government welfare V_t to $\tilde{V}_t$, where

$$\tilde{V}_t(1-s; 1-s_h) = V_t(1-s; 1-s_h)/\left(\gamma y\right).$$

Thus, the government solves the following problem:

$$\tilde{V}_t(1-s; 1-s_h) = \max_b \frac{\underline{S}_{t+1}(b) - s}{s_h - s}\left(1-b\right) + \frac{s_h - \underline{S}_{t+1}(b)}{s_h - s}\beta\tilde{V}_{t+1}\left(1 - \underline{S}_{t+1}(b); 1-s_h\right). \tag{23}$$

The optimal strategy B_t is normalized to $\tilde{B}_t$ accordingly:

$$\tilde{B}_t(1-s; 1-s_h) = B_t(1-s; 1-s_h)/\left(\gamma y\right).$$

Thus, to prove equation (20) and (21) is equivalent to prove the following two equations, for any $\lambda \in (0, 1/(1-s_h))$,

$$\tilde{V}_t(\lambda(1-s); \lambda(1-s_h)) = \lambda\tilde{V}_t\left(1-s; 1-s_h\right) \tag{24}$$

$$1 - \tilde{B}_t(\lambda(1-s); \lambda(1-s_h)) = \lambda\left[1 - \tilde{B}_t\left(1-s; 1-s_h\right)\right]. \tag{25}$$

In period T, the governmet's strategy is $\tilde{B}_T(1-s; 1-s_h) = s_h$, and his welfare is given by $\tilde{V}_T(1-s; 1-s_h) = 1-s_h$. If the bank rejects proposal b at period $T-1$, the government updates his belief according to $1 - \underline{S}_T(1-b; 1-s_h) = \left[(1-b) - \beta(1-s_h)\right]/(1-\beta)$. Hence, equation (22), (24) and (25) are satisfied, i.e., the homogeneity holds for period T.

Given the optimal strategy and the belief in period T, we solve the problem in period $T-1$. The solutions for $\tilde{V}_{T-1}$, $\tilde{B}_{T-1}$ and $\underline{S}_{T-1}$ are as follows:

$$\tilde{V}_{T-1}(1-s;1-s_h) = \begin{cases} 1-s_h & \text{if } 1-s \leq 2(1-s_h) \\ \frac{(1-\beta)(1-s)^2+4\beta(1-s_h)(1-s)-4\beta(1-s_h)^2}{4(1-s)-4(1-s_h)} & \text{if } 2(1-s_h) < 1-s \leq 4(1-s_h) \\ \frac{(2-\beta)(1-s_h)(1-s)-(4-3\beta)(1-s_h)^2}{(1-s)-(1-s_h)} & \text{if } 1-s > 4(1-s_h) \end{cases}$$

$$1-\tilde{B}_{T-1}(1-s;1-s_h) = \begin{cases} 1-s_h & \text{if } 1-s \leq 2(1-s_h) \\ (1-\beta)(1-s)/2+\beta(1-s_h) & \text{if } 2(1-s_h) < 1-s \leq 4(1-s_h) \\ (2-\beta)(1-s_h) & \text{if } 1-s > 4(1-s_h) \end{cases}$$

$$1-\underline{S}_{T-1}(1-b;1-s_h) = \begin{cases} \frac{1}{1-\beta}\left((1-b)-\beta(1-s_h)\right) & \text{if } 1-b \leq (2-\beta)(1-s_h) \\ \frac{2}{(1-\beta)(2+\beta)}\left((1-b)-\beta^2(1-s_h)\right) & \text{if } (2-\beta)(1-s_h) < 1-b \leq 1-\bar{b}_T \\ \frac{1}{1-\beta}\left((1-b)-\beta(2-\beta)(1-s_h)\right) & \text{if } 1-b > 1-\bar{b}_T \end{cases}$$

where $1-\bar{b}_T = (4-2\beta-\beta^2)(1-s_h)$. Clearly, the homogeneity holds for period $T-1$.

We now assume that equation (22), (24), and (25) hold for period $n+1$, and prove that they also hold for period n. Define the probability function Λ_n as follows:

$$\Lambda_n(1-s,1-b;1-s_h) = \frac{(1-s)-\left(1-\underline{S}_{n+1}((1-b);(1-s_h))\right)}{(1-s)-(1-s_h)}.$$

Clearly, Λ_n is homogenous of degree zero in its arguments given the homogeneity property of $\underline{S}_{n+1}$. Using the homogeneity of $\tilde{V}_{n+1}$ and Λ_n, we rewrite $\tilde{V}_n$ as, for any $\lambda \in (0, 1/(1-s_h))$,

$$\begin{aligned} \tilde{V}_n(1-s;1-s_h) &= (1/\lambda)\max_{1-\tilde{b}} \quad \{\Lambda_n(1-\tilde{s},1-\tilde{b};1-\tilde{s}_h)(1-\tilde{b}) \\ &+ (1-\Lambda_n(1-\tilde{s},1-\tilde{b};1-\tilde{s}_h))\beta\tilde{V}_{n+1}(1-\underline{S}_{n+1}(1-\tilde{b};1-\tilde{s}_h);1-\tilde{s}_h)\}, \end{aligned}$$

where $1-\tilde{s}_h \equiv \lambda(1-s_h)$, $1-\tilde{s} \equiv \lambda(1-s)$, and $1-\tilde{b} \equiv \lambda(1-b)$. Therefore, we have

$$\tilde{V}_n(1-s;1-s_h) = (1/\lambda)\,\tilde{V}_n(\lambda(1-s);\lambda(1-s_h)),$$

which gives equation (24). The homogeneity of the optimal strategy $\tilde{B}_n$ and the belief function $\underline{S}_n$ easily follows. ***Q.E.D.***

Lemma A.2. *In any period n, the cutoff belief $\hat{s}_n$ is a linear function of s_h with a slope depending on the discount factor β, i.e.*

$$1-\hat{s}_n = g_n(\beta)(1-s_h), \text{ with } g_n(\beta) > 1. \tag{26}$$

Proof: Under the belief $\hat{s}_n$, the government is indifferent between ending the game in period n or period $n+1$. That is, $\tilde{V}_n(1-\hat{s}_n;1-s_h) = \tilde{V}_{n+1}(1-\hat{s}_n;1-s_h)$. According to the homogeneity of $\tilde{V}_n$ and $\tilde{V}_{n+1}$, we have

$$\tilde{V}_n\left(1;\frac{1-s_h}{1-\hat{s}_n}\right) = \tilde{V}_{n+1}\left(1;\frac{1-s_h}{1-\hat{s}_n}\right).$$

This implies that the ratio of $(1 - s_h)$ and $(1 - \hat{s}_n)$ only depends on the underlying parameter β. We summarize this result with $1 - \hat{s}_n = g_n(\beta)(1 - s_h)$, and clearly $g_n(\beta) > 1$. ***Q.E.D.***

Proof of Proposition 2: We need to prove that the maximum negotiation length increases with the degree of information friction Ψ. Let's consider two intervals $[s_l^1, s_h^1]$ with Ψ^1 and $[s_l^2, s_h^2]$ with Ψ^2. Assume $\Psi^1 \leq \Psi^2$. To compare the maximum renegotiation length, we normalize the interval $[s_l^1, s_h^1]$ to $[\tilde{s}_l^1, s_h^2]$ with $1 - \tilde{s}_l^1 = (1 - s_h^2)(1 - s_l^1)/(1 - s_h^1)$. According to the homogeneity properties in Lemma A.1 and Lemma A.2, interval $[s_l^1, s_h^1]$ and interval $[\tilde{s}_l^1, s_h^2]$ have the same maximum renegotiation length. It is easy to see that $\tilde{s}_l^1 \geq s_l^2$ since $\Psi^1 \leq \Psi^2$, the maximum renegotiation length is shorter under interval $[\tilde{s}_l^1, s_h^2]$ than under interval $[s_l^2, s_h^2]$. Thus, the maximum negotiation length is shorter under $[s_l^1, s_h^1]$ than under $[s_l^2, s_h^2]$. ***Q.E.D.***

Proof of Proposition 3:

We need to prove the uniqueness of the monotonic perfect Bayesian equilibrium. We start by establishing the monotonicity of the creditors' welfare in Lemma A.3.

Lemma A.3. *Given the prior that s is uniformly distributed on $[s_l, s_h]$, the creditors' welfare increases with the realization of the reservation value s.*

Proof: For a given prior $[s_l, s_h]$, the welfare of the creditors of reservation s is defined as,

$$W^N(s) = \sum_{t=1}^{T(s)-1} \beta^{t-1} s + \beta^{T(s)-1} \frac{b_{T(s)}}{1-\beta}, \tag{27}$$

where $T(s)$ denotes the period in which the creditors with reservation s accept the offer, given by

$$T(s) = \min\{t : s \leq c_{t+1}\},$$

where c_{t+1} is the equilibrium belief of the government when his optimal proposal is rejected at period t.

For any s_1 and s_2 such that $s_1 < s_2$ and $T(s_1) = T(s_2)$, clearly $W^N(s_1) \leq W^N(s_2)$. For any s_1 and s_2 such that $s_1 < s_2$ and $T(s_1) = T(s_2) - 1$, the difference between $W^N(s_2)$ and $W^N(s_2)$ is

$$W^N(s_2) - W^N(s_1) = \sum_{t=1}^{T(s_1)-1} \beta^{t-1}(s_2 - s_1) + \beta^{T(s_1)-1} s_2 + \beta^{T(s_1)-1} \frac{\beta b_{T(s_1)+1} - b_{T(s_1)}}{1-\beta}. \tag{28}$$

By the definition of $c_{T(s_2)}$, the creditors with reservation $c_{T(s_2)}$ are indifferent between accepting the period-$T(s_1)$ offer and the period-$T(s_2)$ offer. This implies that

$$b_{T(s_1)} = (1-\beta) c_{T(s_2)} + \beta b_{T(s_1)+1}.$$

Substituting the above relation into equation (28), we show that

$$W^N(s_2) - W^N(s_1) = \sum_{t=1}^{T(s_1)-1} \beta^{t-1}(s_2 - s_1) + \beta^{T(s_1)-1}(s_2 - c_{T(s_1)+1}).$$

Because the creditors with reservation s_2 will accept the proposal at period $T(s_2)$, we have $s_2 \geq c_{T(s_2)}$. As a result, we again prove $W^N(s_1) \leq W^N(s_2)$. Given the generality of $T(s_1)$, we essentially proved that $W^N(\cdot)$ weakly increases in s. ***Q.E.D.***

Proof of Proposition 3: We prove this proposition in two steps. First, taken the monotonic trading strategy as given, we show that there exists a unique cutoff signal $\hat{z}$ and a market price p. Given $(\hat{z}, p)$, the government has the unique optimal strategy and beliefs, and the creditors have the unique optimal strategy in the negotiation stage. Second, we show that given the optimal strategies and the beliefs in the negotiation stage, the monotonic trading strategy is optimal for the creditors in the trading stage.

Suppose all the creditors follow the monotonic trading strategy, i.e., selling bonds whenever their signals below the cutoff level $\hat{z}(p)$. This implies that for each realization of (s, η), there exists a unique $\hat{z}(p)$ that clears the market, i.e.,

$$\hat{z}(p) = s - \sigma_z \sigma_\eta \eta. \tag{29}$$

The market price p equals to the expected payoff of the critical creditor $\hat{z}(p)$ by solving equation (11). Given the cutoff signal $\hat{z}(p)$ and the market price p, the government's and the creditors' negotiation strategies are uniquely pinned down according to Proposition 1. This implies that the critical creditor has a unique expected payoff, which pins down a unique p.

Next we check whether the monotonic trading strategy is optimal for the creditors at the trading stage. The expected payoff of a creditor with signal z from the negotiation stage is $E_s[W^N(s, \hat{z}(p))|\hat{z}(p), z]$. The creditor will sell if and only if $p \geq E_s[W^N(s, \hat{z}(p))|\hat{z}(p), z]$. According to Lemma A.3, we have $E_s[W^N(s, \hat{z}(p))|\hat{z}(p), z]$ weakly increases with the signal since a higher signal implies that s is likely to be higher. Therefore, the monotonic trading strategy is optimal. ***Q.E.D.***

Proof of Proposition 4:

After observing the secondary market price p, the government uses the market clearing condition (29) and updates his belief about s to $[s_l^G, s_h^G]$, where $s_l^G = \max\{\hat{z}(p), s_l\}$ and $s_h^G = \min\{\hat{z}(p) + \sigma_z \sigma_\eta, s_h\}$. Clearly, the updated belief $[s_l^G, s_h^G]$ is a subset of the prior belief $[s_l, s_h]$, since $s_l^G \geq s_l$ and $s_h^G \leq s_h$. According to Proposition 2, both a lower s_h and a higher s_l shorten the maximum negotiation length. ***Q.E.D.***